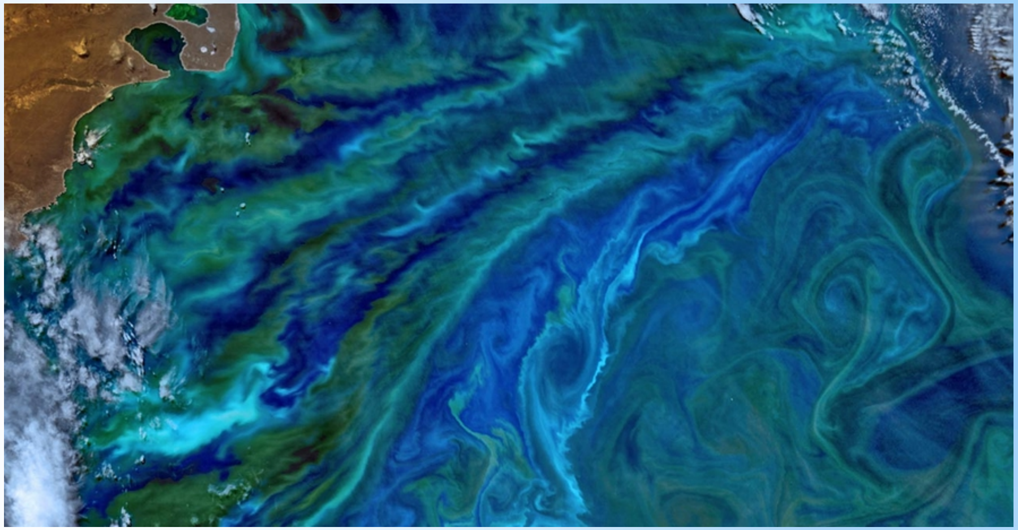
MMSLS, June 2015

chaotic *mixing*



Phytoplankton bloom, off the Atlantic coast of South America, NASA

Ruty Mundel, Erick Fredj, Hezi Gildor *and Vered Rom-Kedar, Physics of Fluids, (**2014**)* **26**,126602 , .

Rotem Aharon, VRK, and Hezi Gildor, "When complexity leads to simplicity: ocean surface mixing simplified by vertical convection", Phys. Fluids 24, 056603 (2012);

D. F. Carlson, E. Fredj, H. Gildor, VRK Deducing an upper bound to the horizontal eddy diffusivity using a stochastic Lagrangian model, Environmental Fluid Mechanics, 1567-7419 (Print) 1573-1510 (Online), 2010

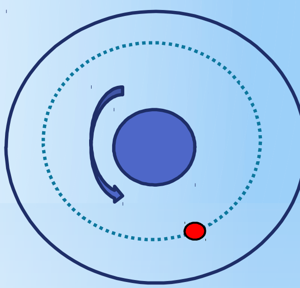
VRK and A.C. Poje Universal properties of chaotic transport in the presence of diffusion. Phys. Fluids 11 (8) (1999) 2044-2057, 1999.

VRK, A. Leonard and S. Wiggins An analytical study of transport, mixing and chaos in an unsteady vortical flow. J. Fluid Mech., 214:347-394, ***1990****.*

Fluid mixing - kinematics

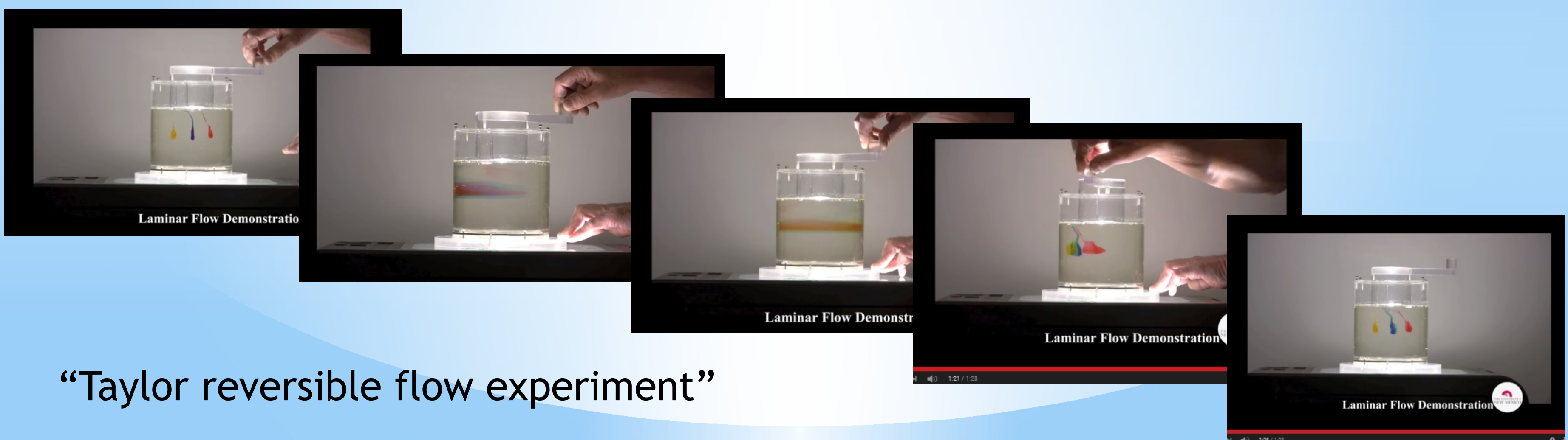
Given an Eulerian velocity field u=u(x,t) how does a passive scalar field c(x,t) mixes?
(dye, pollutant, plankton, algae, proteins, temperature, density, vorticity, cells(?))

- If u=0 : by molecular diffusion $c_t = D\Delta c$



- Else : the advection-diffusion equation:

$$C_t + U(x,t)\cdot\nabla C = D\Delta C,$$



"Taylor reversible flow experiment"

Fluid mixing

The advection diffusion equation:

$$C_t + U(x,t)\cdot\nabla C = D\Delta C,$$

For D=0 C is constant along characteristics (particles paths):

$$\frac{dx}{dt} = U(x,t)$$

Efficient stirring = chaotic particle paths !

U(x,t) can be steady 2d or 3d, time periodic, quasi-periodic, a-periodic, chaotic or turbulent,

depending on the **Reynolds number** and on the **body forces**

Main Biological applications: small scales – Stokes flows with unsteady forces by boundaries

ocean / lakes - Large scale flows + eddy diffusivity on smaller scales

Mixing problems: from micro mixers through traditional lab-scales/mechanics

:to geophysics (see reviews by T. Tel (06), Ottino & Wiggins (04),.., Prants (2013))

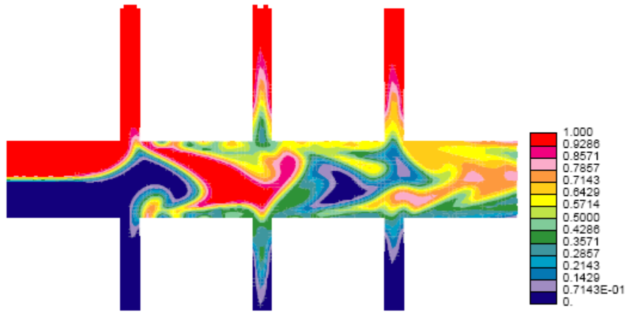


Fig. 2. (Color online.) Concentration distribution of a passive scalar (red: full concentration, dark blue: zero concentration) in a micromixer. The flow in the main channel is stationary and is manipulated by time-periodic flows in the secondary channels. The frequency of these perturbations can be used to enhance mixing. Picture by S.D. Müller, I. Mezić, J.H. Walther and P. Koumoutsakos, with their kind permission.

.Micromixer – Muller et al

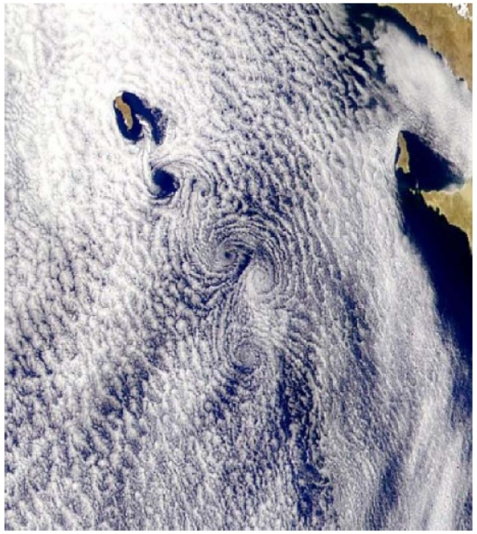


Fig. 10. (Color online.) Cloud patterns around Guadalupe Island, August 20, 1999 (NASA SEAWIFS image).

Clouds over
,Guadalupe island
NASA

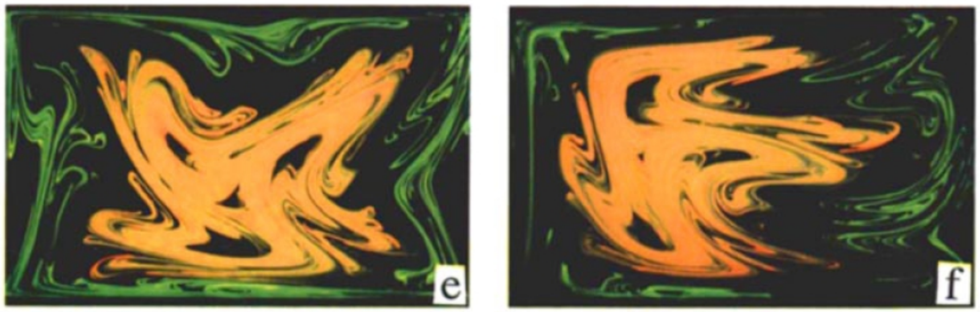


Figure 5 The stretching and folding mechanism characteristic of chaotic mixing is clearly demonstrated by this experiment, which shows the time evolution of two passive blobs in a time-periodic cavity flow. The top figure shows the initial conditions. After a few periods, one of the blobs undergoes significant stretching, whereas the other remains trapped in a regular island (Leong 1989).

Leong – lab mixing of viscous fluid

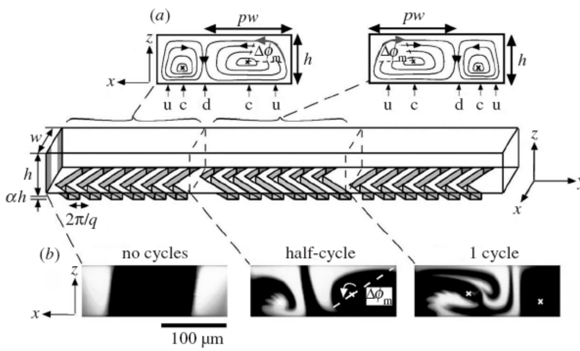


Figure 7. Closed streamlines in the cross-section of each half-cycle. (*a*) The mixer; (*b*) flow visualizations in the cross-section. (Reproduced with permission from Stroock *et al.* (2002). Copyright (2002) AAAS.)

Micro-mixers Stroock et al 02

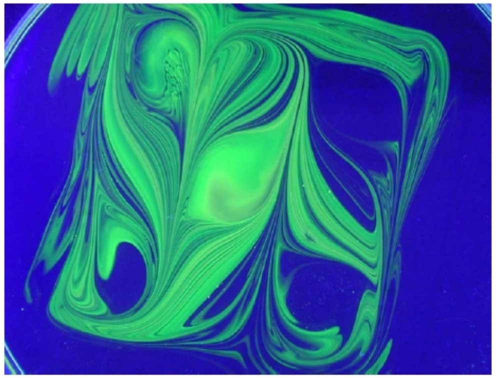


Fig. 3. Shape of a dye (flouresceine) droplet (of initial diameter about 1 cm) after stirring on the surface of a thin layer of glycerol in a Petri dish. The stirring protocol is that of the experiment by Villermaux and Duplat [214]: a number of parallel cuts is made by a rod through the fluid in two direction in an alternating manner. Experiment carried out by I.M. Jánosi, K.G. Szabó, T. Tél, and M. Wells at the von Kármán Laboratory of Eötvös University, Budapest.

Petri dish mixing Janosi et al 07

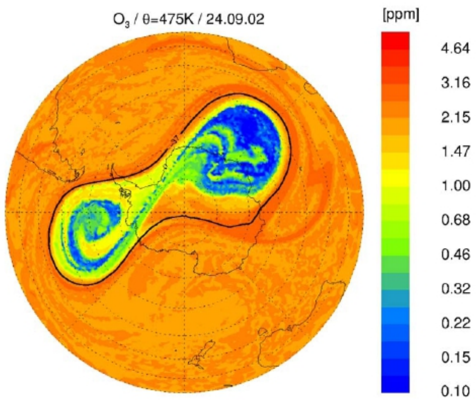


Fig. 5. (Color online.) Ozone distribution (in mixing ratio measured in parts per million (ppm)) above the South Pole region at about 18 km altitude on September 24, 2002. The chemical reactions leading to ozone depletion are simulated over 23 days in the flow given by meteorological wind analyses (from Grooß, Konopka, and Müller, Ozone chemistry during the 2002 Antarctic vortex split [58], with permission) during a very rare event when the so-called polar vortex splits into two parts.

, Ozone vortex split, September 02
Groob et al.

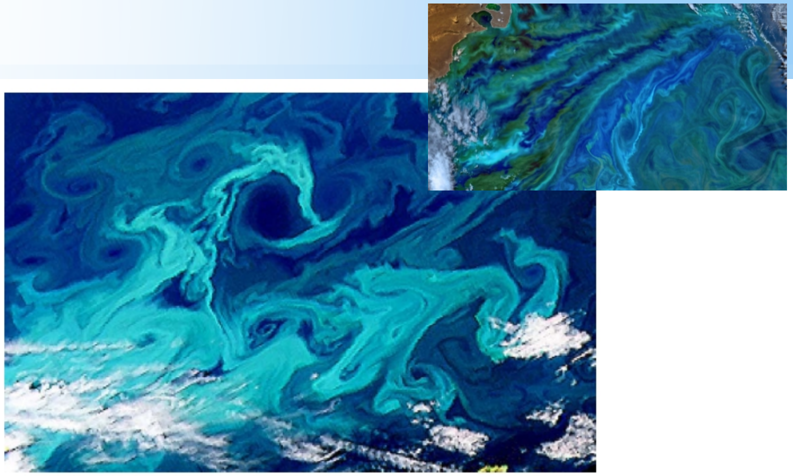


Fig. 4. (Color online.) Filamentation in a phytoplankton bloom in the Norwegian Sea. Provided by the SeaWiFS Project, NASA/Goddard Space Flight Center, and ORBIMAGE, URL: http://visibleearth.nasa.gov/cgi-bin/viewrecord?5278.

Phytoplankton bloom in the Norwegian Sea
SeaWiFS, NASA

Mixing is a complex phenomena:

What can we learn about this complex phenomena from simple models?

Zvi Artstein: Reality is a good approximation of mathematical models

yet:

Need to get the appropriate simple models…

:Content

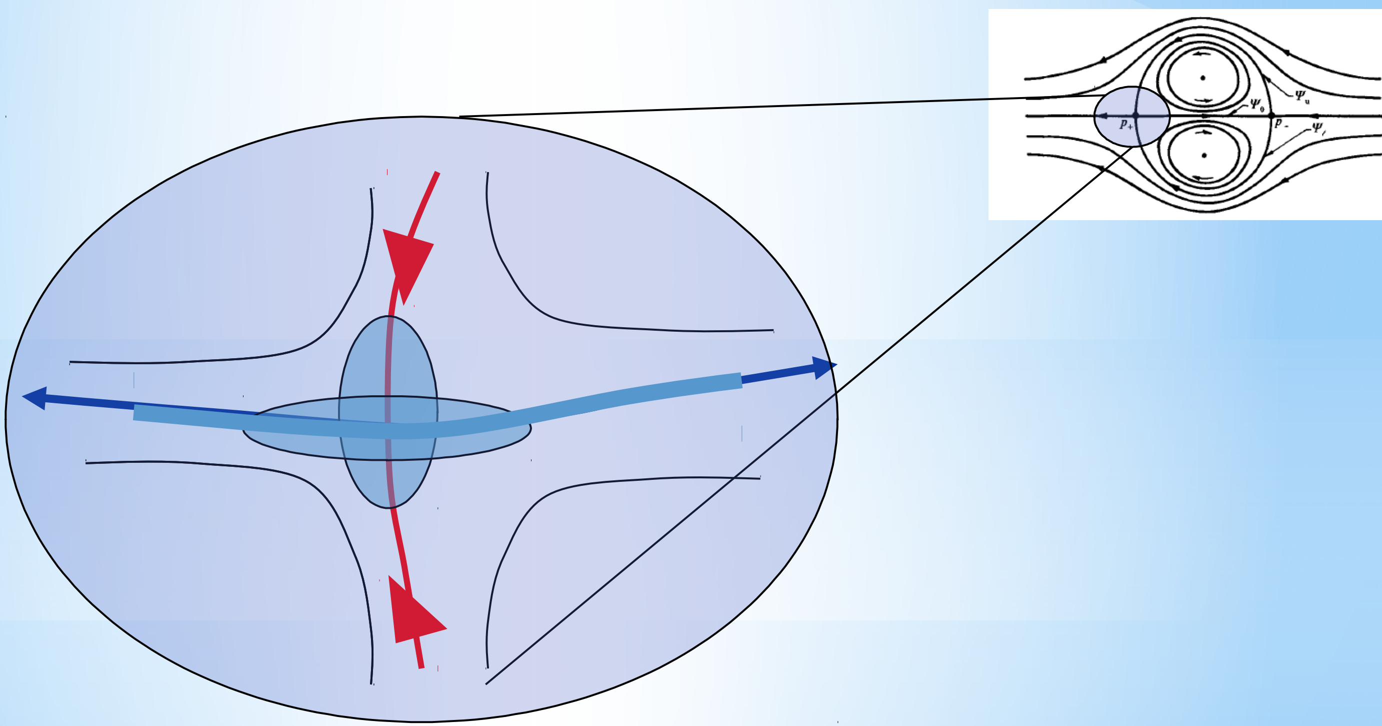
Some mathematical principles governing fluid mixing

A new diagnostic for identifying structures in an unsteady flow

Principles governing chaotic Mixing

* Chaotic Mixing appears only in unsteady flows/viscous 3d flows
* **The unstable manifold is the observable structure in many flow visualizations**
* **The stable and unstable manifolds and their generalizations to LCS govern the transport for many initial value problems**
* There are DS scales (that differ from the traditional fluid-mechanics scales) associated with homoclinic tangles and these determine the transport
* Three dimensionality may simplify surface motion
* Combining visualization and velocity data may be used to supply bounds on eddy diffusivity
* New simple diagnostic tools for identifying coherent structures, transport and dividing surfaces

* Unstable manifold "attracts" passive scalars:



Unstable (stable) manifold of a solution $x_0(t)$: the collection of all initial particle positions that asymptote to $x_0(t)$ exponentially in backward (forward) time

The unstable manifold* is the observable structure in many flow visualizations

layer upstream of $W^u_{-,\epsilon}$, see figure 14. The above arguments apply to a broad class of flows having similar structure, namely hyperbolic stagnation points with cyclic motion near them, implying that the unstable manifold is the observed structure in many flow visualizations, depending on how the fluid is marked. In many ways the manifold acts as an attractor although it cannot be in the usual sense because of incompressibility.

RK ,Leonard, Wiggins **JFM 1990**

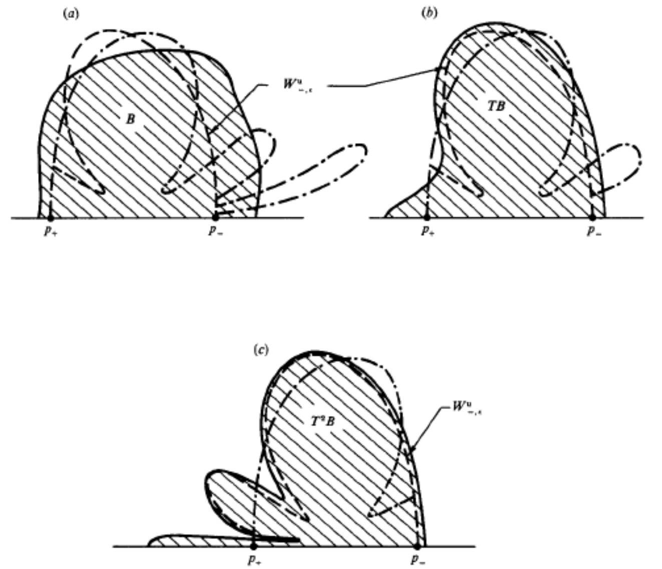


FIGURE 14. The motion of a general initial shape B; (a) $t = 0$, (b) $t = 2\pi\gamma$, (c) $t = 4\pi\gamma$.

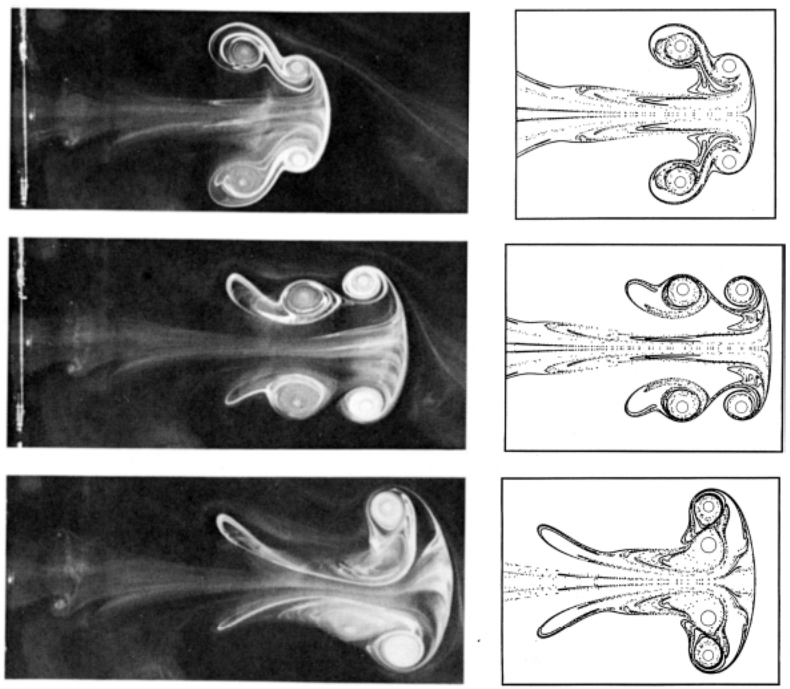
FIGURE 15. Flow visualization of a turbulent vortex ring. (A. Glezer & D. Coles 1987, private communication.)

Unstable manifold of the OVP

Unstable manifold for axis symetric vortex rings:

Shariff, Leonard and Ferziger 89,06:

:Leap-frogging rings



IG. 10. Unstable manifold at different phases for two leapfrogging rings. Photographs are from Yamada and Matsui (Ref. 11).

Flow isulization:Yamada and Matsui, 78

Ring in a shock tube

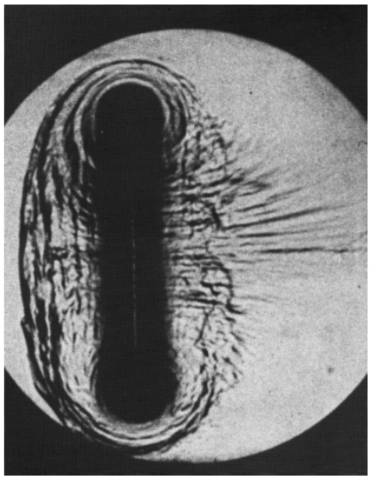


FIG. 7. Schlieren visualization of a shock tube generated ring propagating to the left. Courtesy of B. Sturtevant.

Schlieren visualization o
a ring in a shock tube,
B. Sturtevant

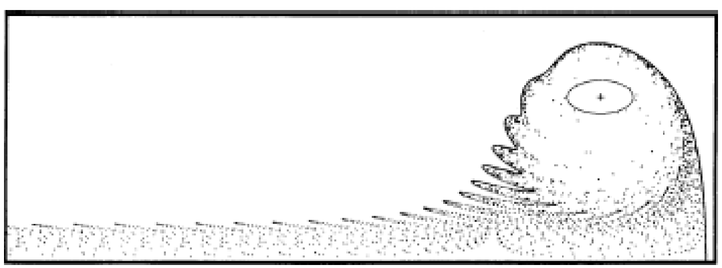
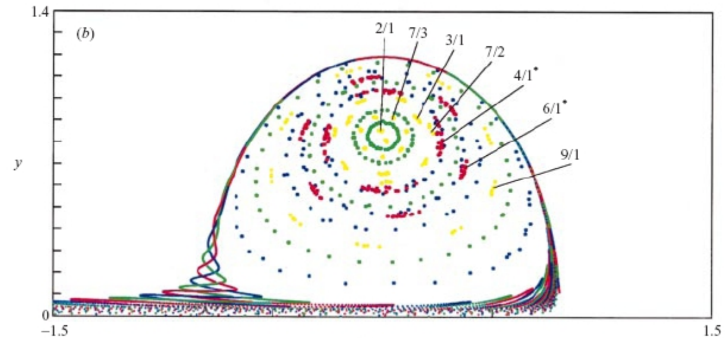


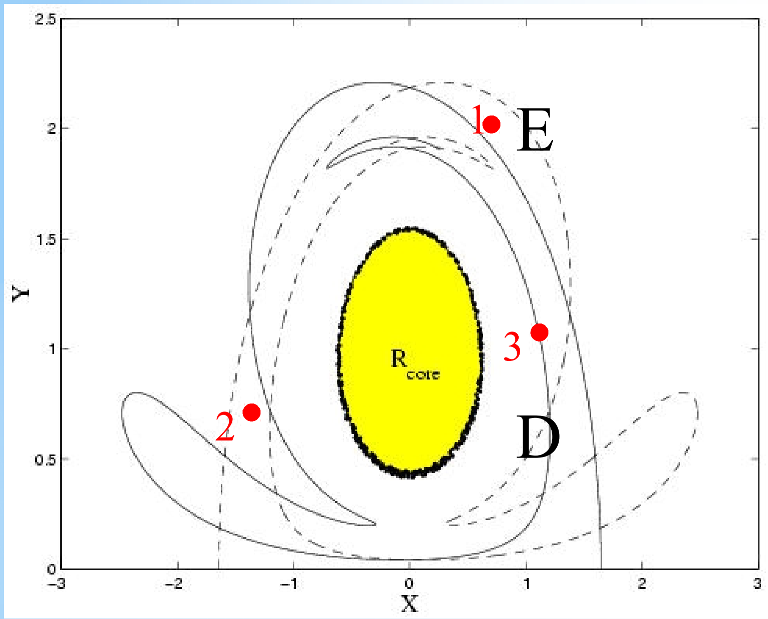
FIG. 2. Unstable manifold of the forward periodic point for an elliptical core ring, $\lambda=2$, $\alpha=(a+b)/(2r_c)=0.15$



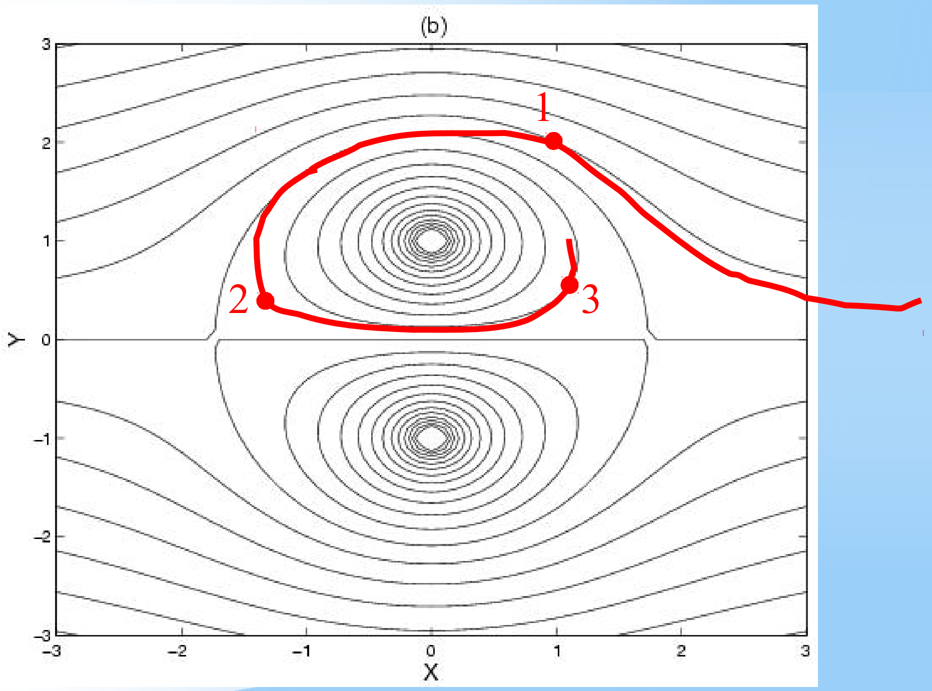
Krasny and Nitsche –
roll-up of a vortex disc to a vortex ring (02)

* The stable and unstable manifolds control the transport of passive scalars:

Flux: Trapped fluid per unit time = $\mu(E)\dfrac{\omega}{2\pi}$



:Poincare map in time sample fluid trajectories every period



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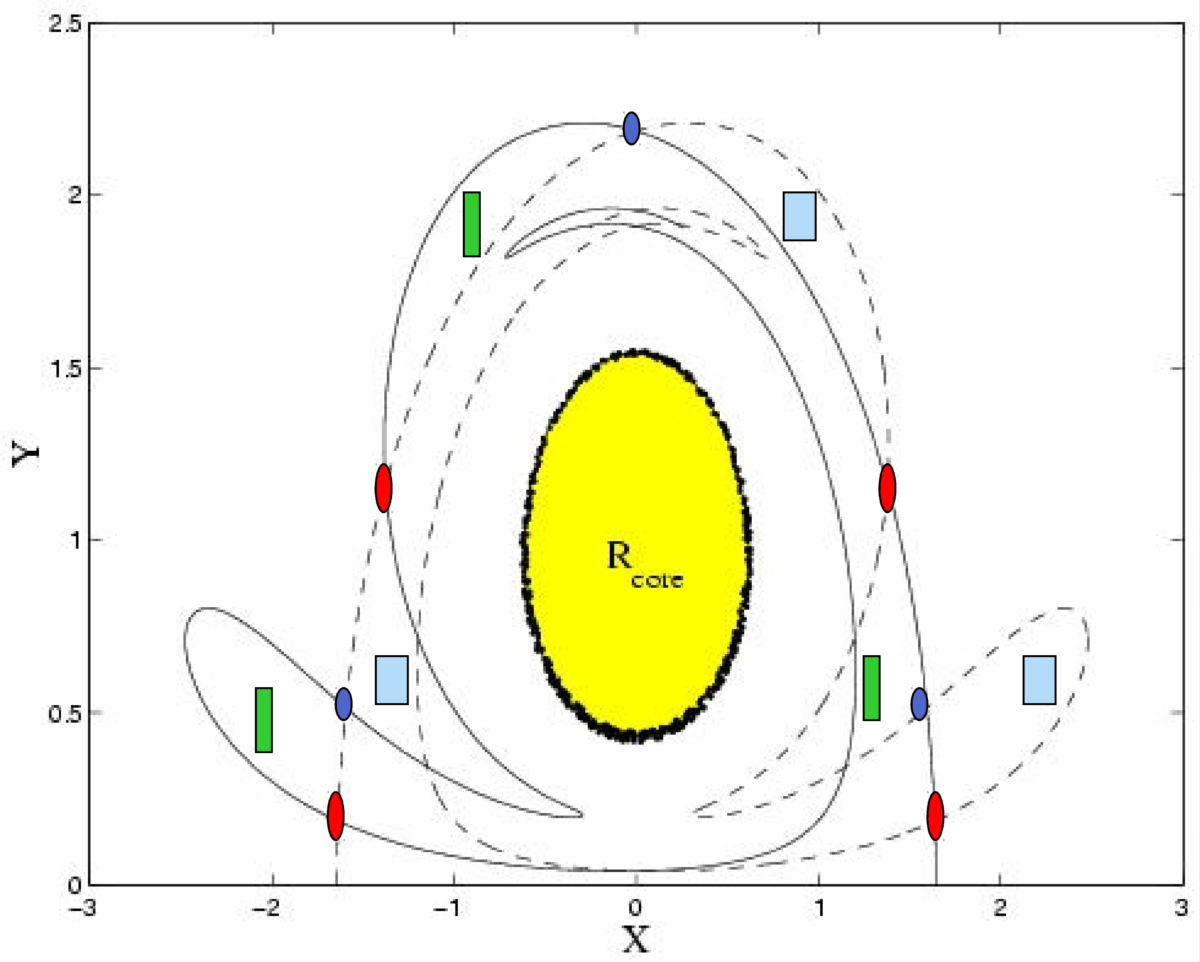
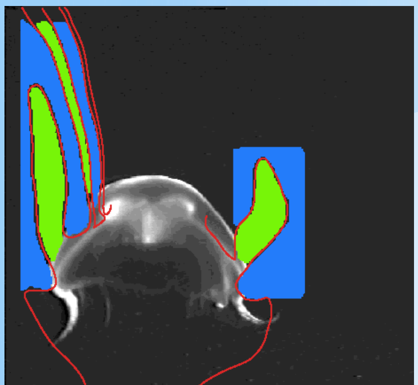


„Turnstile” flux mechanism (Channon and Lebowitz 80, MacKay et al 84“
)Fluid transport – RK et al 89

Many examples showing “lobe” mechanism in numerical/experimental settings (some of these include diffusion/noise/other weak effects!)



Dabiri et al (07)

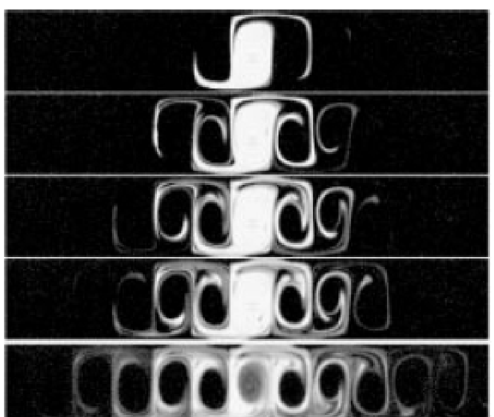


FIG. 2. Sequence of images showing the mixing of uranine dye; oscillation amplitude $b = 0.12$. The images (starting from the top) are taken 18, 39, 57, 75, and 189 s (≈1, 2, 3, 4, and 10 oscillation periods) after the time dependence has been turned on.

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of Lobes in Chaotic Mixing of Miscible and Immiscible Impurities

T. H. Solomon, S. Tomas, and J. L. Warner

Department of Physics, Bucknell University, Lewisburg, Pennsylvania 17837

Principles governing chaotic Mixi

2D STEADY FLOWS ARE "TOO SIMPLE"

TIME PERIODIC FLOWS ARE "GOOD SIMPLE" MODELS
FOR MANY REAL LIFE MIXING PROBLEMS

* Chaotic Mixing appears only in uns ... visco
* The uns ... in many flow visualizations
* Th ... manifolds and their generalizations to LCS govern the ... any initial value problems
* There are DS scales (that differ from the traditional fluid-mechanics scales) associated with homoclinic tangles and these determine the transport
* Three dimensionality may simplify surface motion
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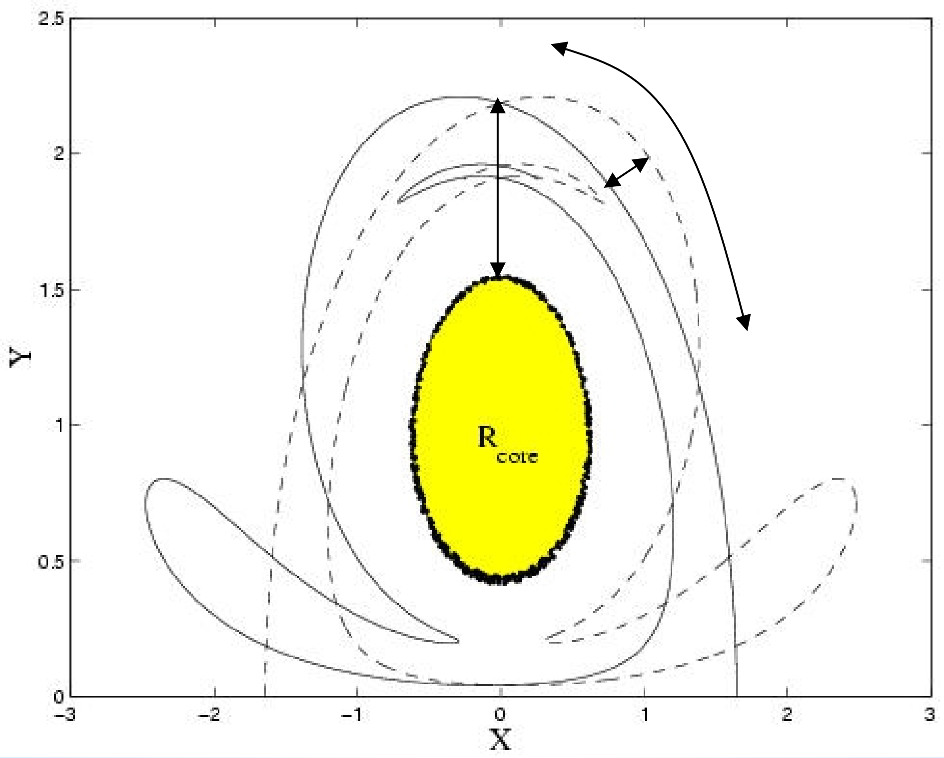
* Homoclinic tangles characteristics:

RK&Poje, 99, RK03

$$u(x,t) = u_0(x;\omega) + Au_1(x,\omega t)$$



Universal flux dependence on frequency:(RK&Poje, 99)

Adiabatic limit (Neishtadt, Kaper, R-K)

+

Fast oscillations (Poincare)

+

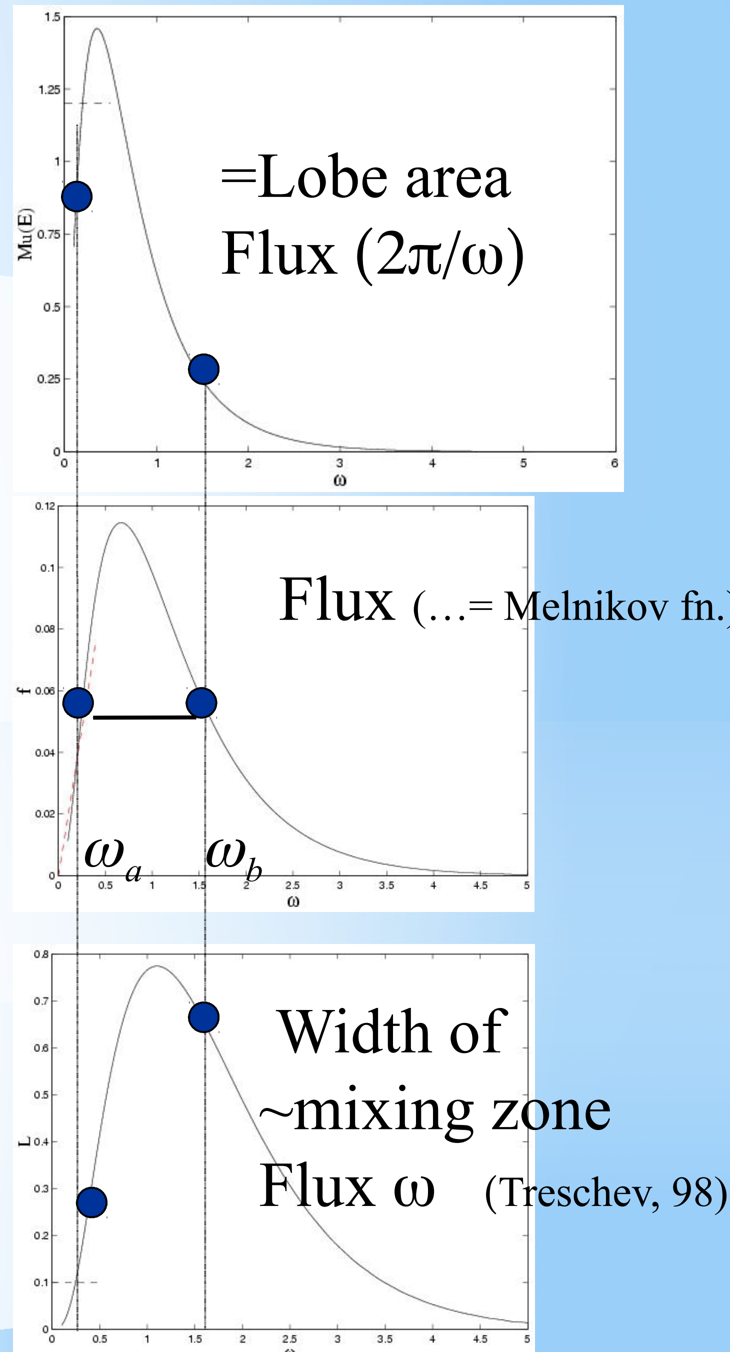
Flux continuity (assume oscillating part is not too **) large

⇓

Flux is always non-monotone

⇓

!Equi-flux frequencies exist



Mixing with equi-flux frequencies and molecular diffusion:

:Initially

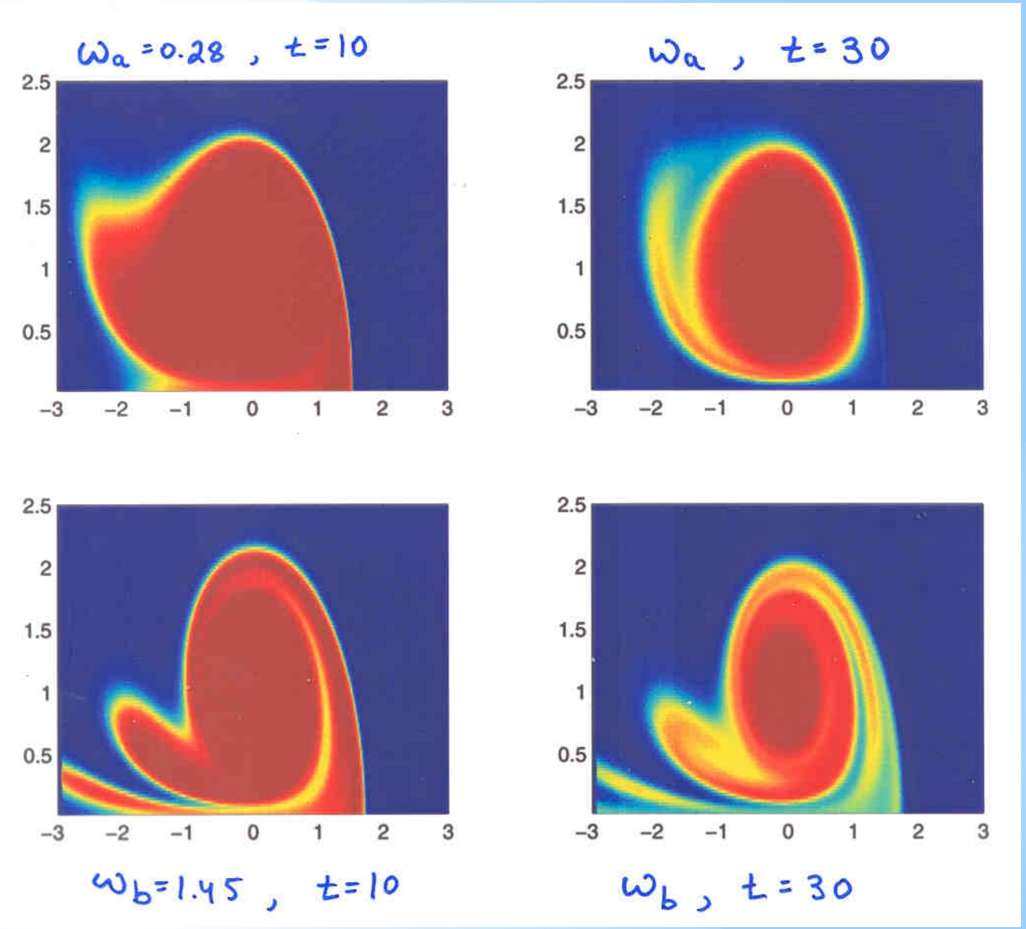
Lower frequency

↓

Stronger gradients

↓

Stronger effect of Diffusion on transport



$$C_t + U(x, y, \omega t) \cdot \nabla C = D \Delta C, \quad D = 0.0001$$

:Later

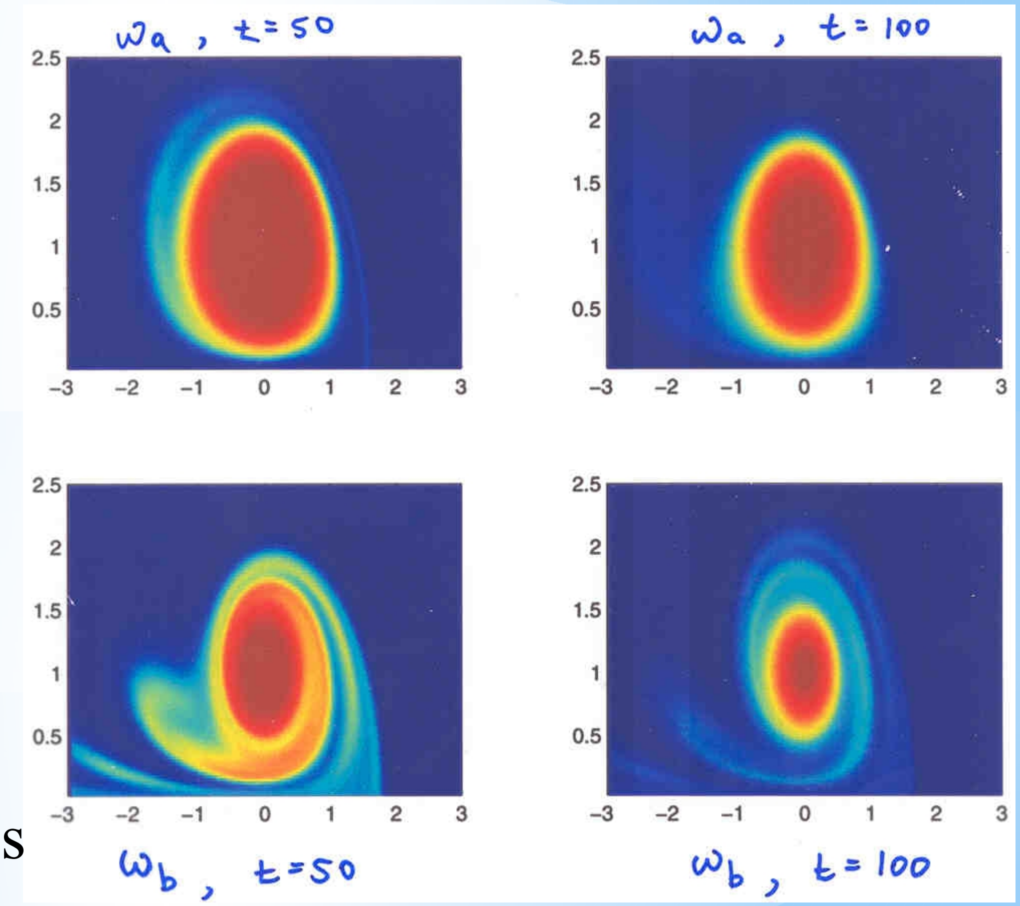
Lower frequency

↓

Larger Core size

↓

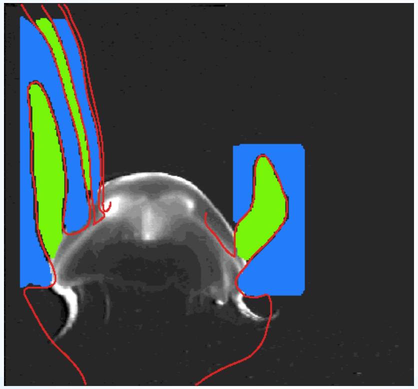
Asymptotic transport is governed by the properties of the Chaotic advection!



$$C_t + U(x, y, \omega t) \cdot \nabla C = D \Delta C, \quad D = 0.0001$$

Challenge: Apply these scaling principles to “real” flows !

-homoclinic scales – should apply to “essentially periodic” flows:



Dabiri et al (07)

- Universality of tangle-scales and diffusion interaction?

- Homoclinic scales near hydrodynamic instabilities?

– Optimization of mixing ? (Balasuriya 2011-15)

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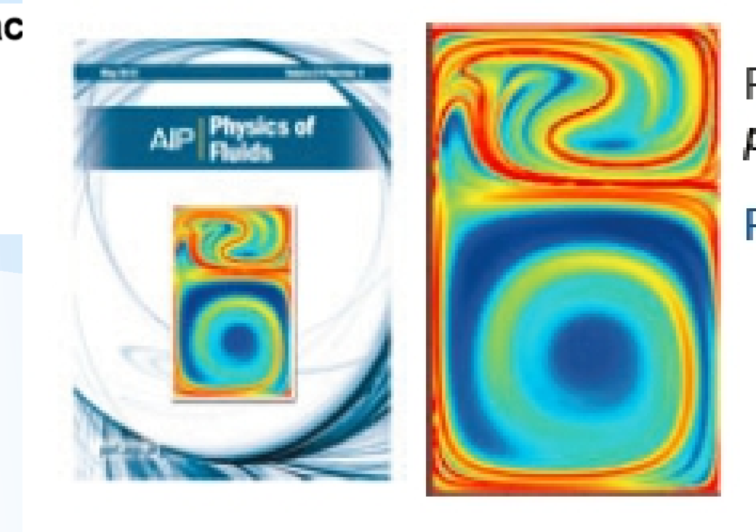
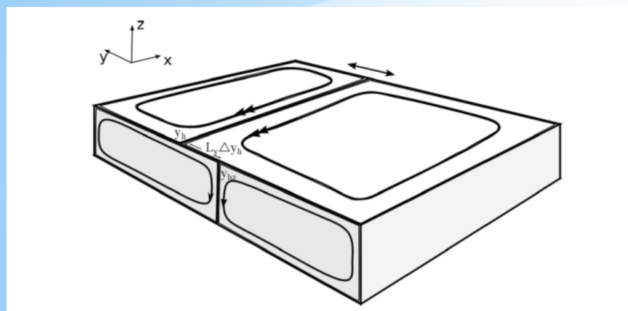
When complexity leads to simplicity: Ocean surfac vertical convection

Rotem Aharon, Vered Rom-Kedar, and Hezi Gildor

Citation: Phys. Fluids 24, 056603 (2012); doi: 10.1063/1.4719147



Consider:

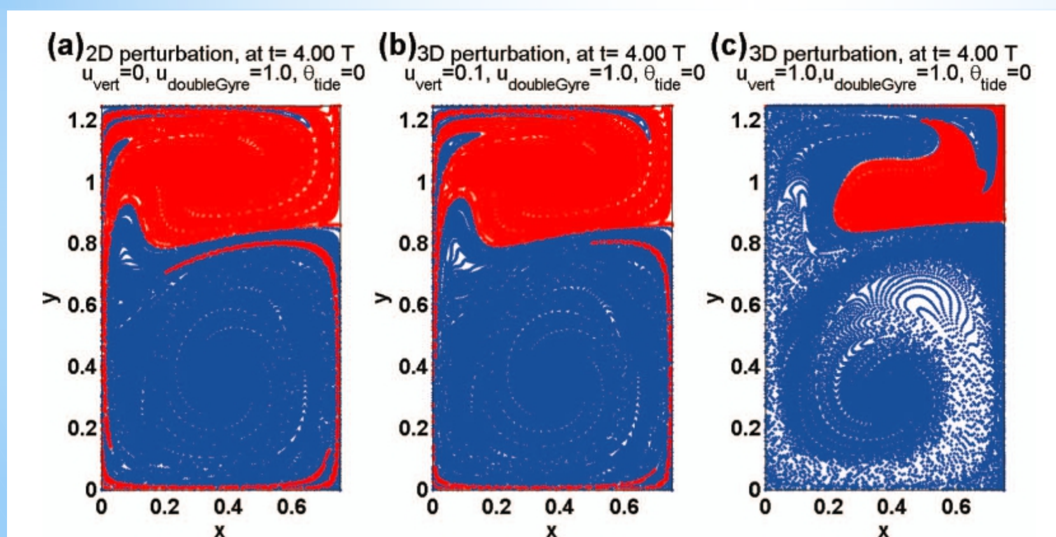


Previous works:

3d simplifies surface particles in turbulent/random flows (get attractors!) Ott, Sommerer, Shumacher, Eckhart and co-workers 1991-1996

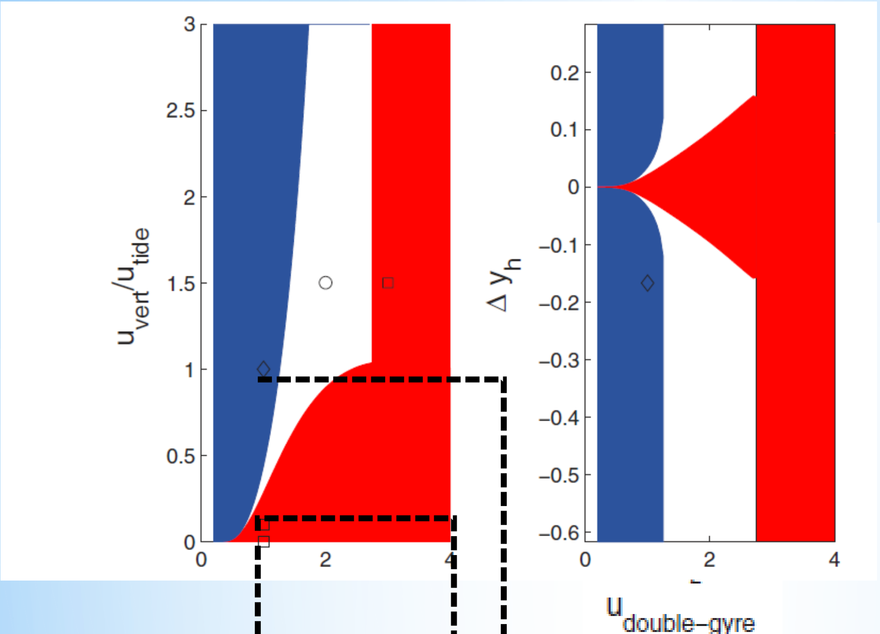
Stirling -2000 – also in rivers' interface to the ocean

Result:



Why? Manifolds.. Averaged surface flow is not area-preserving!

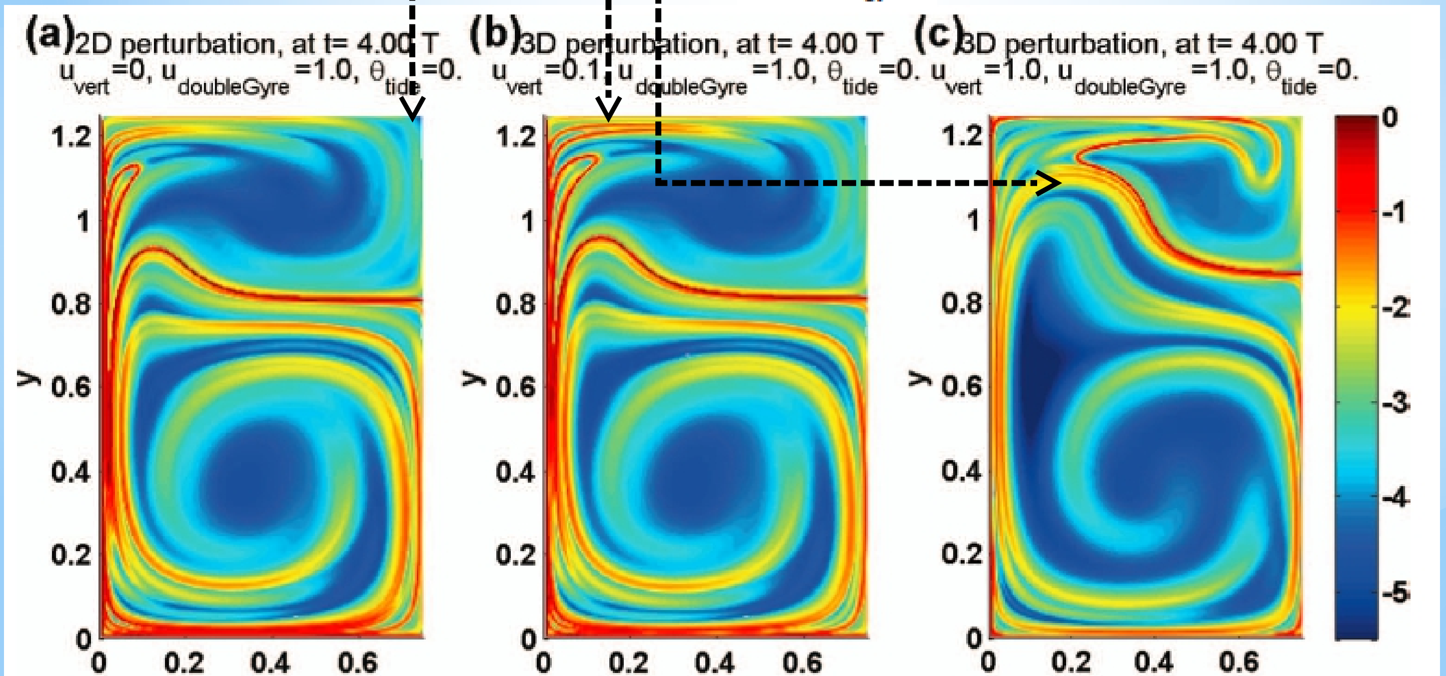
Why? Manifolds..: Theoretically, by Melnikov function:



* homoclinic tangles

* unidirectional flux

* phase dependent tangle



RD fields

Open question: can you apply this principle to “real” flows?

-combination of homoclinic scales and 3d “averaged” effect?

T. Tél et al. / Physics Reports 413 (2005) 91–196 95

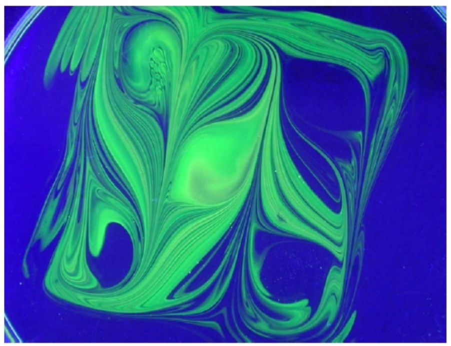


Fig. 3. Shape of a dye (flouresceine) droplet (of initial diameter about 1 cm) after stirring on the surface of a thin layer of glycerol in a Petri dish. The stirring protocol is that of the experiment by Villermaux and Duplat [214]: a number of parallel cuts is made by a rod through the fluid in two direction in an alternating manner. Experiment carried out by I.M. Jánosi, K.G. Szabó, T. Tél, and M. Wells at the von Kármán Laboratory of Eötvös University, Budapest.

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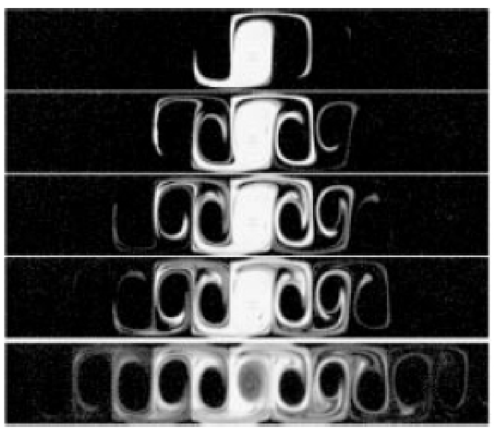

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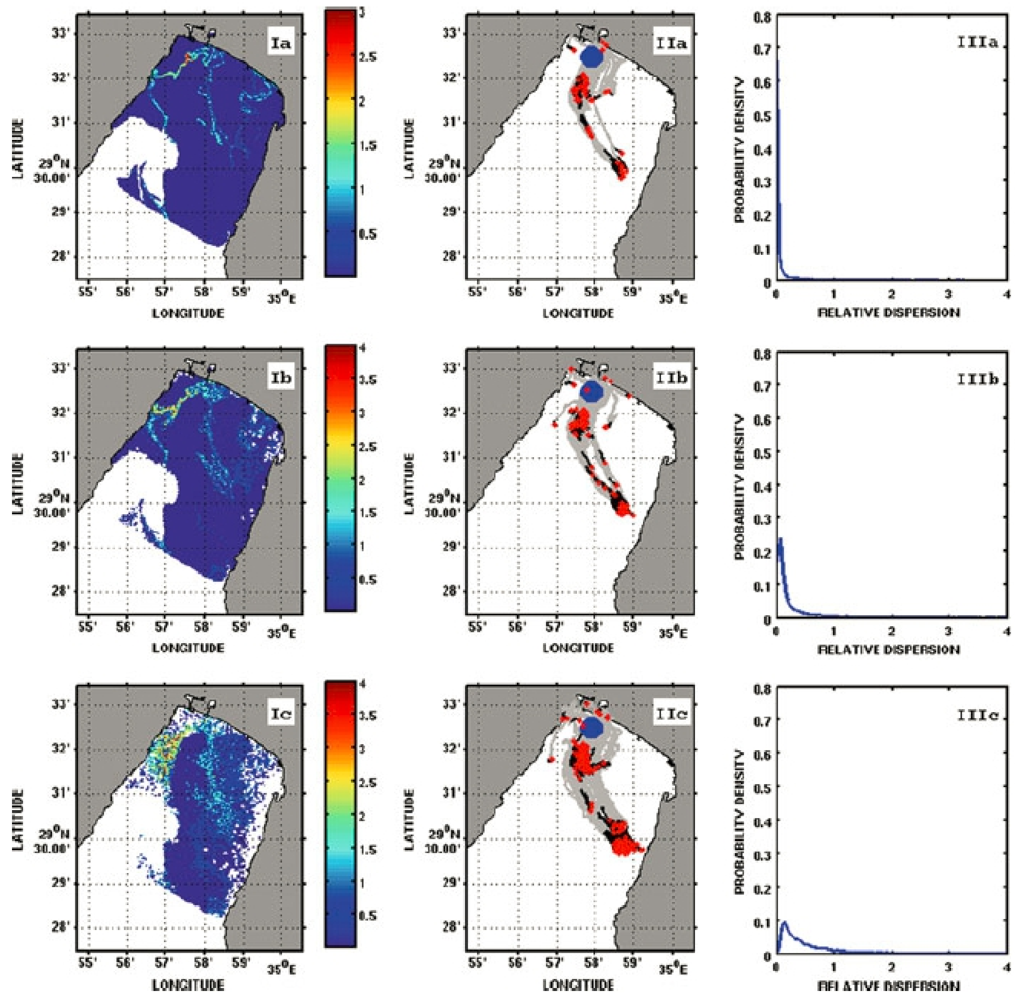
* Three dimensionality may simplify surface motion
* Combining visualization and velocity data may be used to supply bounds on eddy diffusivity (or motility?)
* New simple diagnostic tools for identifying coherent structures, transport and dividing surfaces

Deducing an upper bound to the horizontal eddy diffusivity using a stochastic Lagrangian model

Daniel F. Carlson · Erick Fredj · Hezi Gildor ·
Vered Rom-Kedar

$$dX(t) = \mathbf{U}_{LS}(\mathbf{X}(t), t)\,dt + \sqrt{2K\,dt}\,dw(t); \quad X(t=0) = x_0$$



LCS + noise destroys dividing surface? ➔ Upper bound on K : 1-10 m^2 / s

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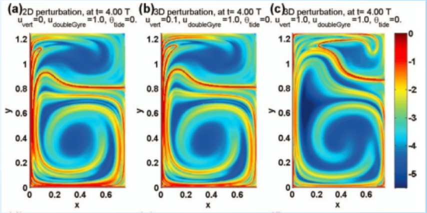
Mixing diagnostics- Lagrangian observables

1) Absolute dispersion (||x(t)-x(0)||
2) Relative dispersion (<||x(t)-y(t)||> where ||x(0)-y(0)||=d<<1)
3) Finite Time Lyapunov Exponents (FTLE)
4) Finite time ergodic measures
5) Arc-length growth
6) Averages of observables along trajectories (e.g. Mesohyperbolicity).
7) Curvatures/other geometrical measures
8) Eigenstates of the transfer operator
9) FLI, UFLI



NEW - Extreme value fields

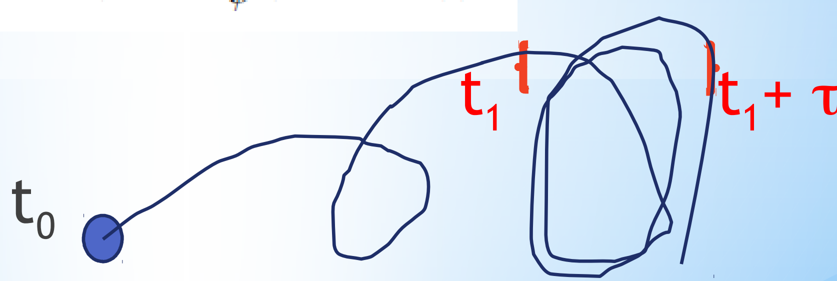
Flow:

$$\frac{dx}{dt} = u(x,t), \qquad x \in \mathbb{R}^n,\ n = 2 \text{ or } 3 \qquad x(t_0;t_0) \;=\; x_0$$

Extreme value fields:

$$\begin{aligned}
M_\phi^+(\tau; x_0, t_1) &= \max_{t\in[t_1,t_1+\tau]}\phi(x(t;t_0)),\\
M_\phi^-(\tau; x_0, t_1) &= \min_{t\in[t_1,t_1+\tau]}\phi(x(t;t_0)),\\
M_\phi^{shift}(\tau; x_0, t_1) &= M_\phi^+(\tau; x_0, t_1) - M_\phi^-(\tau; x_0, t_1),\\
M_\phi^{mean}(\tau; x_0, t_1) &= \frac{1}{2}(M_\phi^+(\tau; x_0, t_1) + M_\phi^-(\tau; x_0, t_1)),
\end{aligned}$$

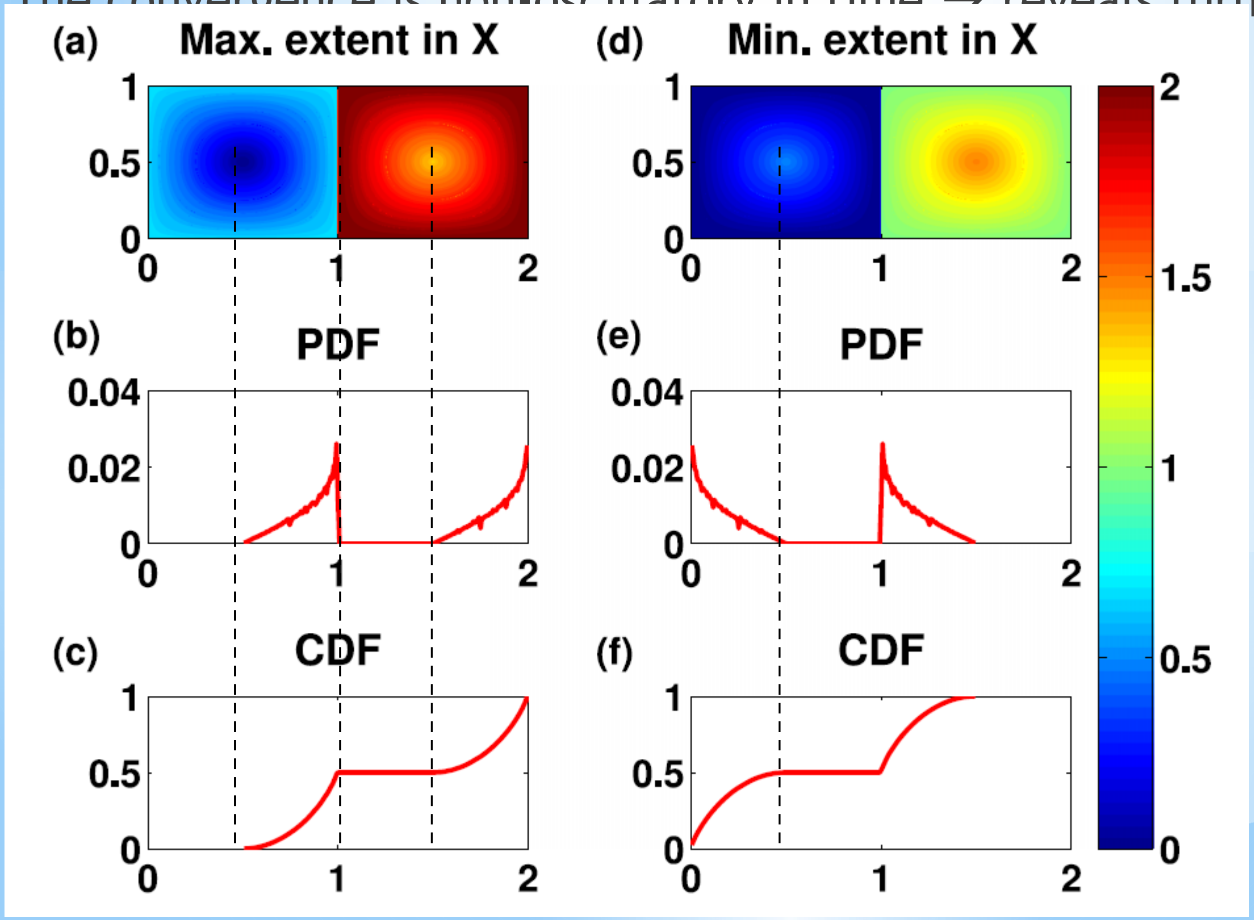
$\phi()$: an observable



Related work: Collet 01, Freitas et al 08,13, Holland et al 12, Lucarini et al. 12

MET for the steady double gyre

- All i.c. in an ergodic component have the same MET value
- Typically, different ergodic components have different $M_r^{+,-}$ values
- CS have concentric MET field ➔ linear PDF ➔ quadratic CDF
- The convergence is non-oscillatory in time → reveals turnover time



MET for the unsteady double gyre

Mixing zone:

Discontinuous at the CS boundary – a nice detection tool !

A delta function in the PDF, a discontinuity in the CDF:

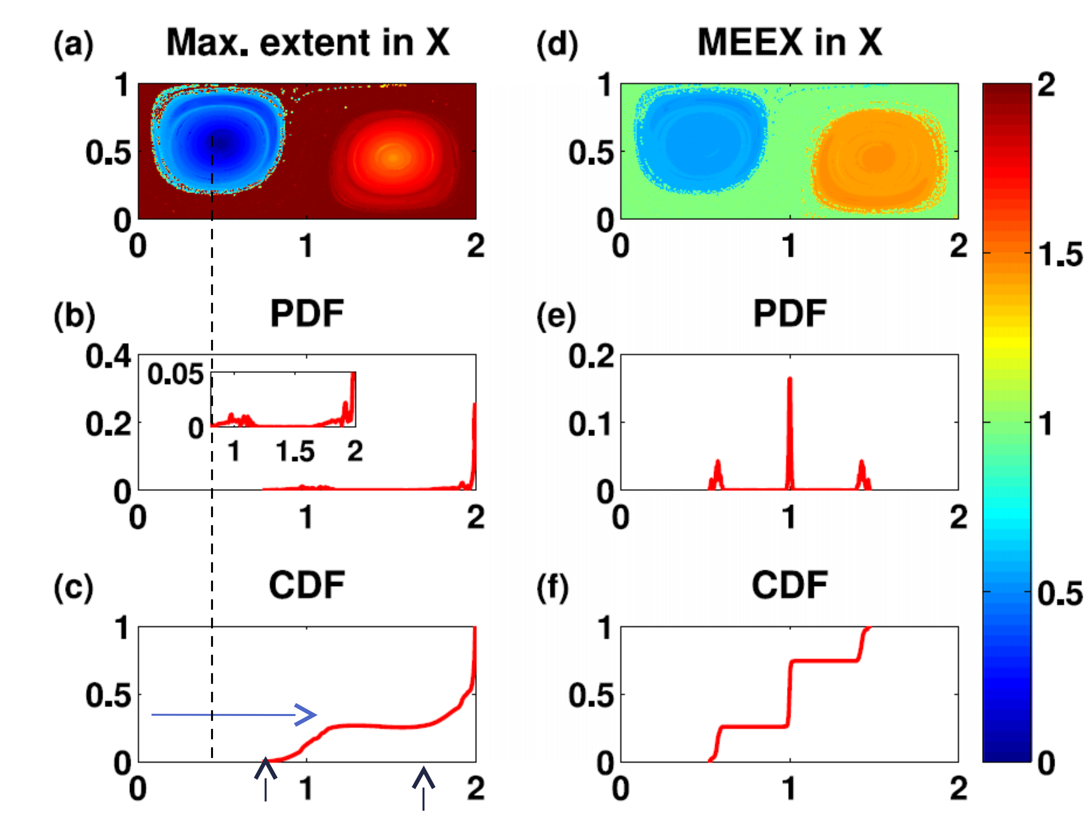
The CS are oscillating :

Center is still well defined

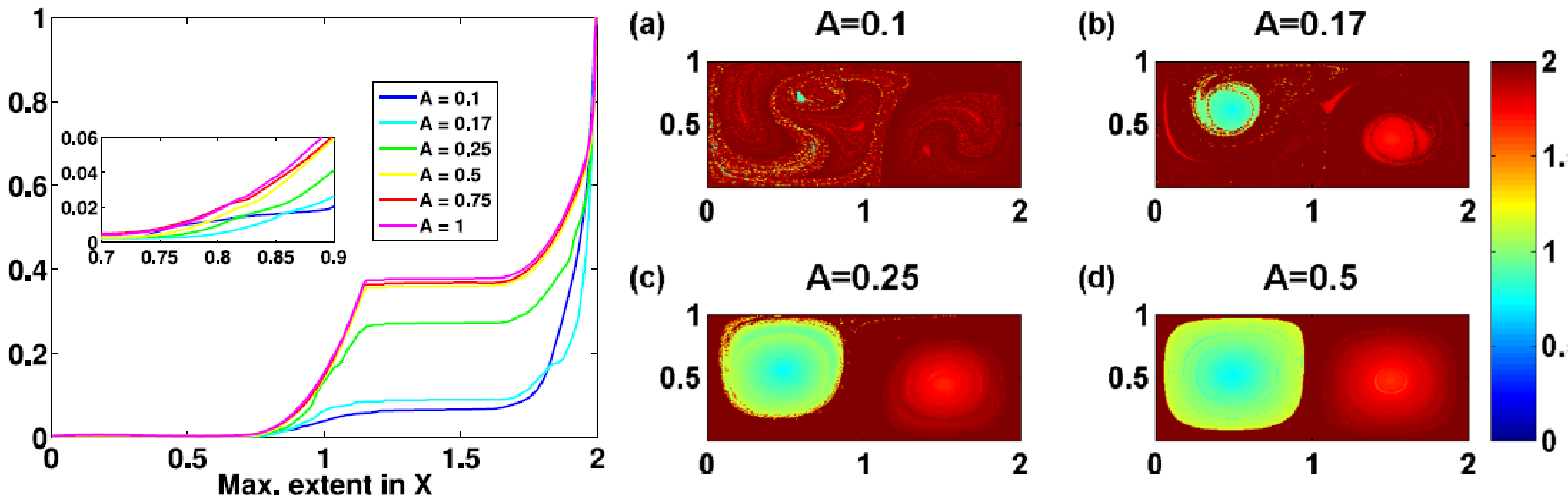
(for each MET?)

Oscillations are detectible

by the mismatch from i.c.



The CDF signatures of CS & mixing zones



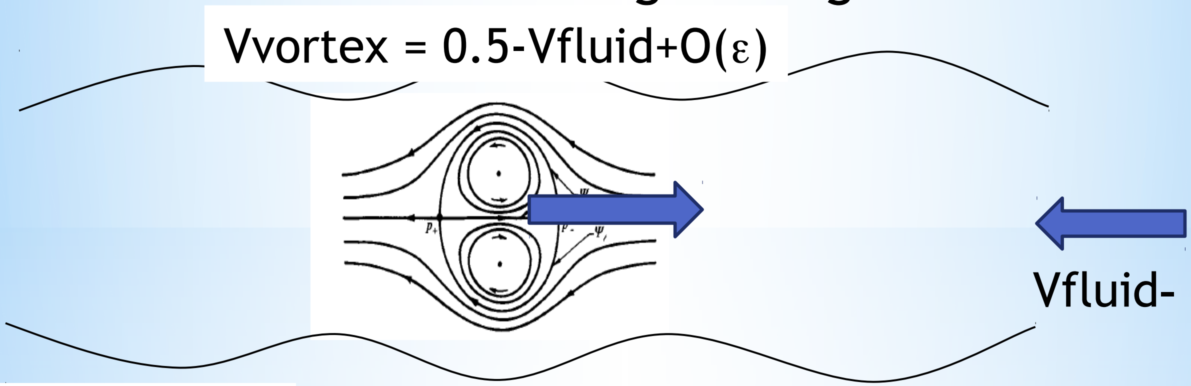
$$\Psi_{xy}(x, y, t) = -A \sin(\pi f(x,t)) \sin \pi y$$
$$f(x,t) = \epsilon \sin \omega t \, x^2 + (1 - 2\epsilon \sin \omega t) x.$$

$\epsilon = 0.25, \omega = 2\pi/10$, $t_1 = 0, \tau = 200$ (20 tidal periods)

MET for open flows

- The METs are well defined
- The CDFs are well defined for a given region of i.c.

OVP:



$$\psi(x,y,t) = -\log \frac{(x-x_v)^2 + (y-y_v)^2}{(x-x_v)^2 + (y+y_v)^2} - v\,y + \epsilon\, x\, y \sin(\omega t)$$

$$\frac{dx_v}{dt} = \frac{1}{2y_v} - v + \epsilon x_v \sin(\omega t),$$

$$\frac{dy_v}{dt} = -\epsilon y_v \sin(\omega t),$$

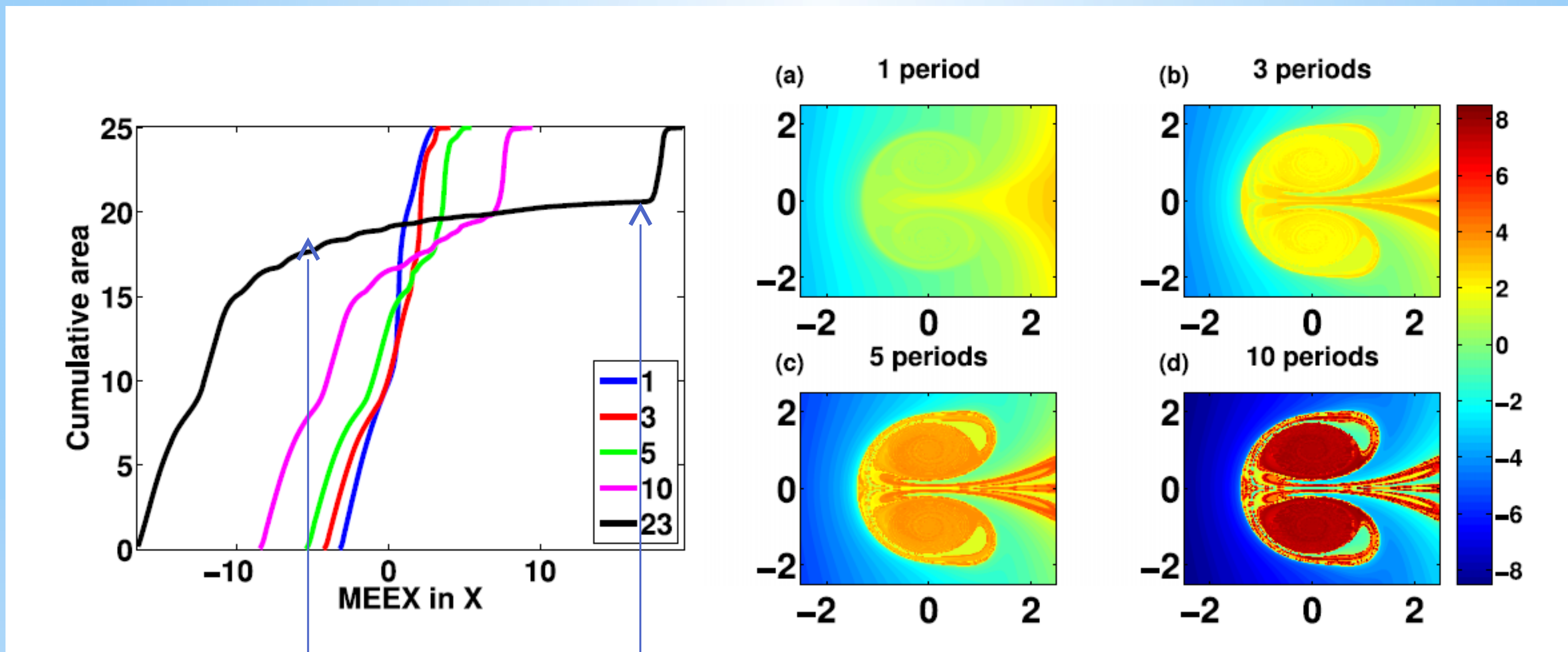
J. Fluid Mech. (1990), *vol.* 214, *pp.* 347–394 347
Printed in Great Britain

An analytical study of transport, mixing and chaos in an unsteady vortical flow

By V. ROM-KEDAR[1],† A. LEONARD[2] AND S. WIGGINS[3]

FIGURE 15. Flow visualization of a turbulent vortex ring. (A. Glezer & D. Coles 1987, private communication.)

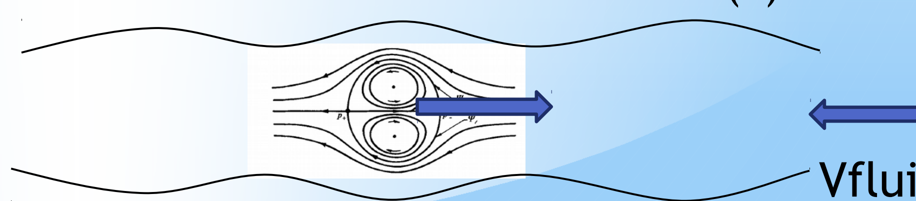
MET for open flows: time dependence



CS location (*.5)

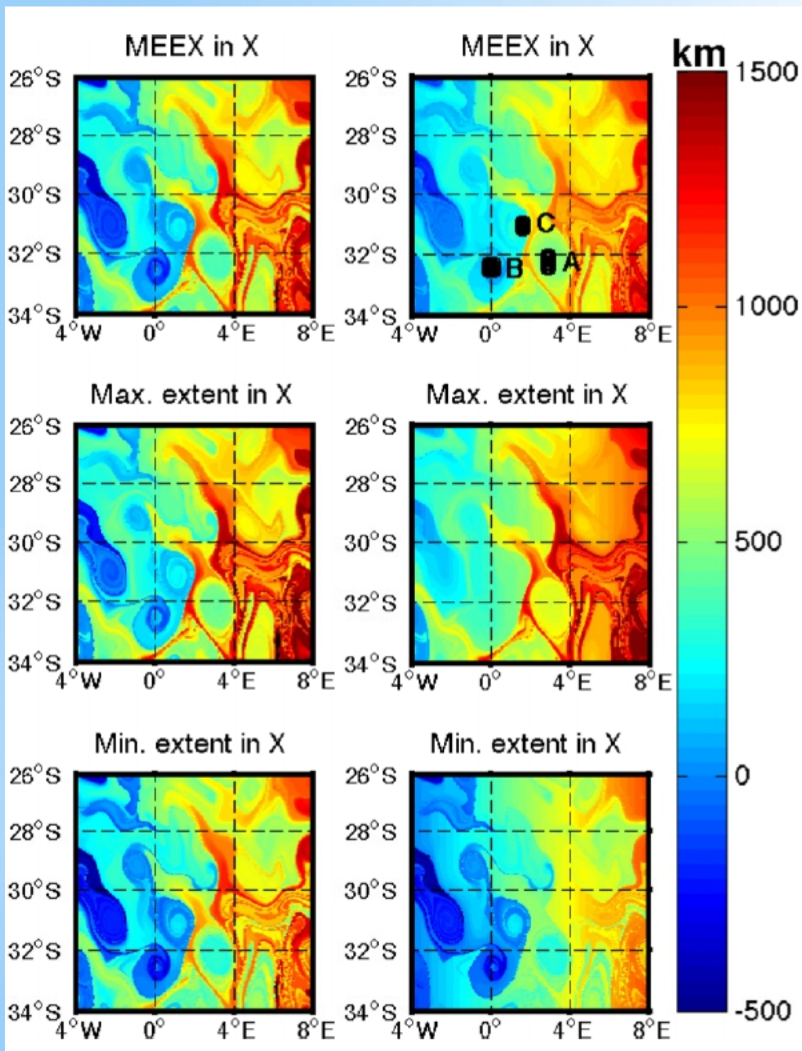
Lobes shedding

Vvortex = 0.5-Vfluid+O(ε)



Vfluid-

:MET for the South Atlantic



60-90 days 0-90 days

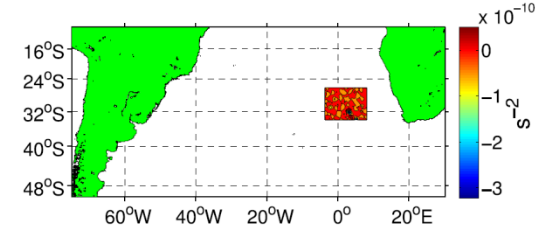
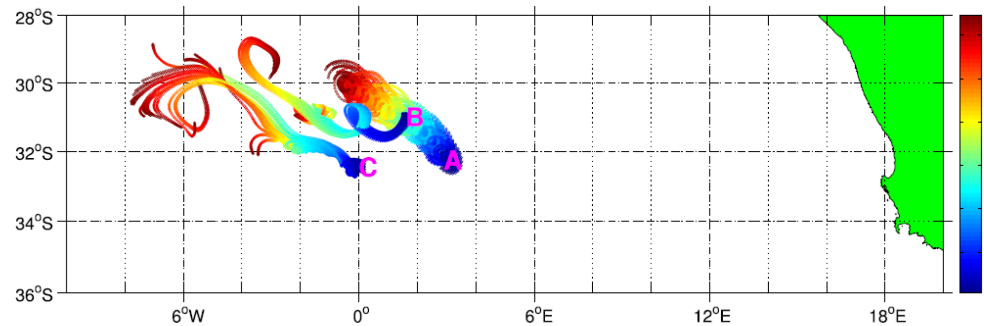


FIG. 13. The South Atlantic region in which particles are seeded on November 24, 2006 is the colored rectangular domain. The colors represent the Okubo-Weiss field (showing some spurious structures[57]) for November 22, 2006, the closest date to November 24, 2006 for which raw data from the AVISO data set are available.



MET & other EV fields → data reduction

Extreme value fields provide new information:

- Converge monotonically in time to a common value on each ergodic component
- Ergodic components belonging to a CS have a **“signature” in the CDF**
- The extremal value is typically discontinuous at the CS-mixing zone boundary
- Properly chosen directions enable one to deduce info from CDF plots

* Examples:
  * Double gyre
  * Oscillating vortex pair
  * The South Atlantic (and Mediterranean, not shown)

Some mathematical principles governing chaotic Mixing

* Chaotic Mixing appears only in unsteady flows
* The unstable manifold is the observable structure in many flow visualizations
* The stable and unstable manifolds and their generalizations to LCS govern the transport for many initial value problems
* There are new scales associated with the homoclinic tangles and these govern the transport (with Poje, 1999, RK 03)
* Three dimensionality may simplify surface motion (with Aharon & Gildor, 2012)
* Combining visualization and velocity data may be used to bound eddy diffusivity (with Carlson, Fredj, Gildor, 2010)
* New simple diagnostic tools for identifying coherent structures [[also - transport and dividing surfaces? Bio-transport in the ocean?]]

  (with Mundel, Fredj, Gildor, 2014)

REFERENCES

Ruty Mundel, Erick Fredj, Hezi Gildor and Vered Rom-Kedar, Physics of Fluids, (2014) **26**,126602 , .

Rotem Aharon, VRK, and Hezi Gildor, "When complexity leads to simplicity: ocean surface mixing simplified by vertical convection", Phys. Fluids 24, 056603 (2012); http://dx.doi.org/10.1063/1.4719147. ON THE COVER.

D. F. Carlson, E. Fredj, H. Gildor, VRK Deducing an upper bound to the horizontal eddy diffusivity using a stochastic Lagrangian model, Environmental Fluid Mechanics, 1567-7419 (Print) 1573-1510 (Online), 2010

VRK and A.C. Poje Universal properties of chaotic transport in the presence of diffusion. Phys. Fluids 11 (8) (1999) 2044-2057, 1999.

VRK, A. Leonard and S. Wiggins An analytical study of transport, mixing and chaos in an unsteady vortical flow. J. Fluid Mech., 214:347-394, 1990.