

# A Bovine Babesiosis Mathematical Model With Dispersion

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## **Bovine Babesiosis (BB)**

(Tick Fever, Cattle Fever, Texas Fever, Piroplasmosis, Redwater)

- BB is a tick-borne parasitic zoonotic disease that causes significant morbidity and mortality in cattle.
- Worldwide, over 1.3 billion bovines are potentially at risk of being infected with BB because of frequent exposure to infected ticks.
- The major economic impact of babesiosis is on the cattle industry. However, infections in other domestic animals, including horses, sheep, goats, pigs, and dogs, assume varying degrees of importance throughout the world.

# Babesia

- BB results from infection by a protozoa in the genus Babesia.
- Several different species of Babesia cause BB.
- Babesia bovis (*B. bovis*), Babesia bigemia (*B. bigemia*) and Babesia divergens (*B. divergens*) are the three most common cause of BB infection.

# **Bovine Babesiosis Distribution**

- BB is found wherever the infected tick vectors exist.
- BB is most common in tropical and subtropical areas.
- B. bovis and B. bigemia are more common in Africa, Asia, Australia, Central and South America.
- *B. divergens* is more common in parts of Europe and North Africa.



#### **Eradication of Bovine Babesios In US**

- In 1906, an eradication program led to the creation a National Cattle Fever Tick Eradication Program in the USA.
- •The program targeted all or part of the following southern US states: Alabama, Arkansas, California, Florida, Georgia, Kentucky, Louisiana, Mississippi, Missouri, North Carolina, Oklahoma, South Carolina, Tennessee, Texas and Virginia.
- •By 1943, the tick eradication program had been declared complete, and all that remains today is a permanent BB quarantine zone along the Rio Grande River in South Texas.
- •The existing BB quarantine zone is an approximately 500-mile long swath of land stretching from Del Rio to Brownsville, Texas, ranging in width from several hundred yards to about 10 miles.
- Prior to movement out of the quarantine zone, all cattle must be inspected for ticks, declared tick-free, and dipped in coumaphos or organophosphate acaricides. When BB ticks are found on cattle, the premise of origin is considered infested and placed under quarantine.
- •Cattle on all adjacent premises are then also inspected and traceback work begins in order to locate and treat any livestock that have left the herd and/or to find the source of livestock added to the herd. 5

# **Bovine Babesiosis Treatment, Reintroduction and control**

- A variety of drugs have been used to treat babesiosis in the past, but only diminazene aceturate and imidocarb dipropionate are still in common use. These drugs are not available in all endemic countries, or their use may be restricted.
- To control BB, most cattle producers practice an integrated approach that includes tick control by chemical means (acaricides); cattle vaccination with attenuated strains of B. bovis, B. bigemina or B. divergens, and treatment of clinically-ill cattle.
- Vaccination using live, attenuated strains of the parasite has been used successfully in a number of countries, including Argentina, Australia, Brazil, Israel, South Africa, and Uruguay. The vaccine is provided in either a chilled or a frozen form. One vaccination produces adequate immunity for the commercial life of the animal; however, vaccine breakdowns have been reported. Several recombinant antigens have been shown experimentally to induce some immunity, but commercial vaccines are not yet available.
- Reintroduction of BB is a significant threat.

## Mathematical Model Focus on Bovine Bovis

- There are important heterogeneities among the three main species of BB, namely B. bigemina, B. bovis and B. divergens.
- For example, among the more common species in Colombia and South Africa, B. bigemia is known to be more pathogenic while B. bovis has wider distribution.
- BB is maintained in cattle populations by asymptomatic carriers that have recovered from the disease.
- Typically, B. bovis persists in cattle for years, but B. bigemina survives only for a few months.
- For simplicity, we will use a susceptible-infected-recovered (SIR) compartmental model with no asymptomatic class to focus primarily on B. bovis. However, by adding the asymptomatic class and making a suitable change of parameters, our model is easily adaptable for studying any of the other main species of BB.

## Aranda Lozeno's 2011 Model

- In 2011, Aranda Lozeno, used a system of ordinary differential equations to model the interactions between the BB tick vector and cattle.
- Aranda Lozeno's model assumptions include 100% vertical BB disease transmission in the bovine population, constant birth rate of tick vector and sedentary tick and bovine populations (no dispersion).
- To study the combined effect of vertical BB transmission (in both tick and cattle populations), seasonality, and migration on BB transmission dynamics, we will introduce an extension of Aranda Lozeno's model based on a system of partial differential equations (PDEs).

## **Epidemic Models With Dispersion**

- In 2007, L. J. S. Allen, B. M. Bolker, Y. Lou and A. L. Nevai, studied a SI epidemic model with variable parameters.
- H. Thieme, in 2009, developed a general framework for defining the basic reproduction number R<sub>0</sub> for infinite dimensional population structure.
- This concept was subsequently used in 2012 by N. K. Vaidya, F.-B. Wang and X. Zou for an avian influenza model.
- This was further extended by Y. Lou and X.-Q. Zhao for a reaction diffusion model with incubation period in the vector population.
- An anthrax epizootic model with migration was considered in 2012 by A. Friedman and A. Yakubu.

## SIR BB Mathematical Model

- We use a non-autonomous system of a continuous-time compartmental model with spatial dispersion to study the bovine babesiosis disease epidemic.
- The total bovine population density per unit area at location x and time t,  $N_{R}(x,t)$ , is divided into three compartments: susceptible,  $S_{B}(x,t)$ , infected,  $I_{B}(x,t)$ , and recovered,  $R_{B}(x,t)$ .
- Susceptible bovine hosts have no BB parasite in their bloodstream, but can become infected with BB after receiving bites from babesiosis infected ticks.
- Infected bovines have the Babesia parasite in their bloodstream, and the ٠ recovered bovine are infected bovines that have been treated for the disease.
- At location x and time t, the tick total population density per unit area, •  $N_{\tau}(x,t)$ , is divided into two compartments: susceptible,  $S_{\tau}(x,t)$ , and infected,  $I_{T}(x,t)$ .
- Susceptible ticks do not carry the disease but may become infected after a • blood meal on infected bovine hosts. 10

#### **BB Mathematical Model Assumptions**

- BB parasite transmission in both bovine and tick populations are assumed to be frequency dependent, rather than density dependent, due to homogeneous mixing.
- We assume that the disease transmission rates in the bovine population,  $\beta_B$ , and in the tick population,  $\beta_T$ , are constants.
- To include conservation of bites in our model, we make the simplifying assumption that the average number of tick-biting rate is equal to the average number of bites that a bovine receives per unit time.

# BB Mathematical Model Assumptions (Continued)

- In the bovine population, we assume vertical disease transmission with very high probability (1-q), where q∈(0,1) is the small probability of "no vertical" transmission of BB.
- Similarly, in the tick population, vertical transmission of the Babesia parasite occurs with probability (1-p), where p∈(0,1) is the small probability of "no vertical" transmission of BB.
- A constant  $\lambda_B$  per day of treated infected bovine population is assumed to have recovered, and a constant  $\alpha_B$  per day of the recovered bovine population can return to the susceptible compartment.

#### BB Mathematical Model Assumptions (Continued)

- Endemic BB shows recurrent annual seasonal patterns. In the Venda district of Limpopo Province in South Africa, for example, BB is mainly transmitted during the rainy season (October-May) when tick numbers are higher.
- To capture the observed seasonal effects in our model, we assume that the tick birth rate is τ-periodic.
- For simplicity, we do not make any distinction between ticks on the ground and ticks on the bovine. Grazing cattle tend to disperse over a given area from high concentration to low concentration in order to feed more effectively. For this reason, we shall include a dispersion term in the dynamics of bovine, with constant coefficient d<sub>B</sub>. We will show later on that increased dispersion decreases the probability of BB prevalence.
- Ticks on cattle disperse with the constant dispersal coefficient of the cattle, while the dispersion of ticks on the ground varies very little seasonally. Consequently, we assume that the tick population is dispersing with a constant dispersal coefficient, d<sub>T</sub>, where d<sub>T</sub> is smaller than d<sub>B.</sub>

#### **BB Mathematical Model Parameters**

Parameter	Definition
$\alpha_B$	Rate of treated bovine
$\beta_B$	Bovine per tick infectivity rate
$\beta_T$	Tick per bovine infectivity rate
$\lambda_B$	Rate of recovered bovine
$\gamma_B$	Bovine death rate
$\gamma_T$	Tick death rate
$\mu_B$	Bovine birth rate
$\mu_T$	Tick birth rate
$d_B$	Bovine diffusion coefficient
$d_T$	Tick diffusion coefficient
p	Probability of "no vertical" transmission in ticks
q	Probability of "no vertical" transmission in bovines

#### **BB Mathematical Model**

$$\frac{\partial S_B}{\partial t} - d_B \Delta S_B = \mu_B (S_B + R_B) + \alpha_B R_B + q \mu_B I_B - \beta_B \frac{S_B}{NB} I_T - \gamma_B S_B, \\
\frac{\partial I_B}{\partial t} - d_B \Delta I_B = (1 - q) \mu_B I_B + \beta_B \frac{S_B}{NB} I_T - (\lambda_B + \gamma_B) I_B, \\
\frac{\partial R_B}{\partial t} - d_B \Delta R_B = \lambda_B I_B - (\alpha_B + \gamma_B) R_B, \\
\frac{\partial S_T}{\partial t} - d_T \Delta S_T = \mu_T (S_T + p I_T) - \beta_T S_T \frac{I_B}{N_B} - \gamma_T S_T, \\
\frac{\partial I_T}{\partial t} - d_T \Delta I_T = (1 - p) \mu_T I_T + \beta_T S_T \frac{I_B}{N_B} - \gamma_T I_T, \\
\frac{\partial N_B}{\partial t} - d_B \Delta N_B = (\mu_B - \gamma_B) N_B, \\
\frac{\partial N_T}{\partial t} - d_T \Delta N_T = (\mu_T - \gamma_T) N_T, \\
N_B (x, t) = S_B (x, t) + I_B (x, t) + R_B (x, t), \\
N_T (x, t) = S_T (x, t) + I_T (x, t),
\end{cases}$$

where  $x = (x_1, x_2)$  varies in a region  $\Omega$ , and

$$\triangle = \sum_{j=1}^{2} \frac{\partial^2}{\partial x_j^2};$$

the model parameters are defined in Table 1.

#### Aranda Lozano 2011 BB Model

When  $\mu_T$  is constant,  $\mu_B = \gamma_B$ ,  $\mu_T = \gamma_T$  and  $q = d_B = d_T = 0$ , Model (2.1) reduces to the following ODE BB model of Aranda Lozano:

$$\frac{dS_B}{dt} = (\alpha_B + \mu_B) R_B - \beta_B \frac{S_B}{N_B} I_T, 
\frac{dI_B}{dt} = \beta_B \frac{S_B}{N_B} I_T - \lambda_B I_B, 
\frac{dR_B}{dt} = \lambda_B I_B - (\alpha_B + \mu_B) R_B, 
\frac{dS_T}{dt} = \mu_T p I_T - \beta_T S_T \frac{I_B}{N_B}, 
\frac{dI_T}{dt} = -p \mu_T I_T + \beta_T S_T \frac{I_B}{N_B}.$$
(2.2)

# P<sup>(L)</sup> and R<sub>0</sub>

Setting

$$\mathcal{P}^{(L)} = \frac{\beta_B \beta_T}{\lambda_B p \mu_T} \tag{2.3}$$

for the case  $\frac{N_T}{N_B} = 1$ , Aranda Lozano proved in [3] that the disease-free equilibrium (DFE) of (2.2) is stable if  $\mathcal{P}^{(L)} < 1$  and unstable if  $\mathcal{P}^{(L)} > 1$ .

The threshold parameter  $\mathcal{P}^{(L)}$  is related to the basic reproduction number,  $\mathcal{R}_0$ , used in epidemiology. For system (2.1) with no dispersion ( $d_B = d_T = 0$ ) and constant  $\mu_T$ .  $\mathcal{R}_0$  is defined as follows:

An average of 1 infected bovine gives birth to  $\frac{(1-q)\mu_B}{(\lambda_B+\gamma_B)}$  infected bovine and infects  $\frac{\beta_T}{(\lambda_B+\gamma_B)}\frac{N_T}{N_B}$  ticks. Similarly, average of 1 infected tick gives birth to  $\frac{(1-p)\mu_T}{\gamma_T}$  infected ticks and infects  $\frac{\beta_B}{\gamma_T}$  bovines.  $\mathcal{R}_0$  is the spectral radius of the matrix

$$\begin{pmatrix} \frac{(1-q)\mu_B}{(\lambda_B+\gamma_B)} & \frac{\beta_B}{\gamma_T} \\ \frac{\beta_T}{(\lambda_B+\gamma_B)} \frac{N_T}{N_B} & \frac{(1-p)\mu_T}{\gamma_T} \end{pmatrix},$$

and the DFE is stable if  $\mathcal{R}_0 < 1$  and unstable if  $\mathcal{R}_0 > 1$ . For more on how  $\mathcal{R}_0$  is computed for models with vertical transmission see [9].

## **Simplified BB Model**

$$\frac{\partial s_B}{\partial t} - d_B \Delta s_B = \rho_B r_B + q \mu_B i_B - \beta_B \frac{N_{T0}(t)}{N_{B0}} s_B i_T, \qquad (3.8)$$

$$\frac{\partial i_B}{\partial t} - d_B \Delta i_B = \beta_B \frac{N_{T0}(t)}{N_{B0}} s_B i_T - (\lambda_B + q\mu_B) i_B, \qquad (3.9)$$

$$\frac{\partial r_B}{\partial t} - d_B \Delta r_B = \lambda_B i_B - \rho_B r_B, \qquad (3.10)$$

$$\frac{\partial s_T}{\partial s_T} = d_A \Delta r_B = (100) + (100$$

$$\frac{\partial s_T}{\partial t} - d_T \Delta s_T = (\mu_T - \gamma_T) s_T + p \mu_T(t) i_T - \beta_T s_T i_B - \frac{N_{T0}}{N_{T0}} s_T, \qquad (3.11)$$

$$\frac{\partial i_T}{\partial t} - d_T \Delta i_T = (\mu_T - \gamma_T) i_T - p \mu_T(t) i_T + \beta_T s_T i_B - \frac{N_{T0}}{N_{T0}} i_T, \qquad (3.12)$$

with the boundary conditions for  $x \in \partial \Omega$  and  $t \ge 0$ :

$$\frac{\partial s_B}{\partial \nu} + \alpha (s_B - 1) = \frac{\partial i_B}{\partial \nu} + \alpha i_B = \frac{\partial r_B}{\partial \nu} + \alpha r_B = 0, \qquad (3.13)$$

and

$$\frac{\partial s_T}{\partial \nu} + \alpha (s_T - 1) = \frac{\partial i_T}{\partial \nu} + \alpha i_T = 0.$$
(3.14)

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## **Proliferation Index**

We introduce the principal eigenpair  $(\lambda_*, \phi)$ :

$$\left.\begin{array}{l} \Delta\phi + \lambda_*\phi = 0 & \text{in }\Omega, \\ \frac{\partial\phi}{\partial\nu} + \alpha\phi = 0 & \text{on }\partial\Omega, \\ \lambda_* > 0, \ \phi(x) > 0 & \text{in }\overline{\Omega}, \end{array}\right\} (4.7)$$

and take

$$1 \le \phi(x) \le \phi_1, \tag{4.8}$$

where  $\phi_1$  is a constant.

For clarity we shall first consider the case where

$$\mu_T(t) = \text{constant} = \mu_T.$$

Then both  $N_{T0}$  and  $N_{B0}$  are constants, and we let

$$N = \frac{N_{T0}}{N_{B0}}.$$
 (4.9).

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## **BB Proliferation Index (continued)**

We define as *proliferation index*,  $\mathcal{P}$ , of the disease model (2.1) and (3.3)-(3.16) the number

$$\mathcal{P} = \frac{N\beta_B\beta_T}{\left(\lambda_B + q\mu_B + d_B\lambda_*\right)\left(p\mu_T + d_T\lambda_*\right)}.$$
(4.10)

#### **BB Extinction Or Persistence**

**Theorem 4.3:** If  $\mathcal{P} < 1$ , then the disease-free equilibrium point, DFE = (1, 0, 0, 1, 0), is globally asymptotically stable.

**Theorem 4.4:** If  $\mathcal{P} > 1$ , then there exist positive numbers  $\delta_1$  and  $\delta_2$  such that if either  $i_B(x, 0)$  is not identically equal to zero or  $i_T(x, 0)$  is not identically equal to zero, then

 $i_B(x,t) \ge \delta_1 \text{ and } i_T(x,t) \ge \delta_2 \text{ for all } x \in \Omega, \ t > \widetilde{T}$  (4.11)

#### **BB** Proliferation Index (continued)

We define as *proliferation index*,  $\mathcal{P}$ , of the disease model (2.1) and (3.3)-(3.16) the number

$$\mathcal{P} = \frac{N\beta_B\beta_T}{\left(\lambda_B + q\mu_B + d_B\lambda_*\right)\left(p\mu_T + d_T\lambda_*\right)}.$$
(4.10)

## **Dispersion and BB Control**

It is interesting to note that if we consider Model (2.1) and (3.3) - (3.7) with  $\mu_T$  constant but without diffusion, then  $d_B = d_T = 0$  and the corresponding proliferation index is

$$\widetilde{\mathcal{P}} = \frac{\beta_B \beta_T}{\left(\lambda_B + q \mu_B\right) p \mu_T},$$

so that  $\mathcal{P} < \widetilde{\mathcal{P}}$  if N = 1. We conclude that dispersion of the bovines and ticks increases the stability of the *DFE* and decreases the probability that initial infection will become endemic. This can be explained by the fact that dispersion of small initial infection does not allow a "critical mass" of infections to settle in the region.

#### **Extension To Periodic Tick Birth Rate**

In this case,

$$X(t) = \left(\begin{array}{c} I_{\mathcal{B}} \\ I_{T} \end{array}\right)$$

satisfies the equation

$$\frac{dX}{dt} = A(t)X \tag{7.1}$$

where

$$A(t) = \begin{pmatrix} -(\lambda_B + q\mu_B + d_B\lambda_*) & N\beta_B \\ \beta_T & -(p\mu_T(t) + d_T\lambda_* + \frac{N'_{T0}(t)}{N_{T0}(t)}) \end{pmatrix}.$$
(7.2)

We write  $X(\tau)$  in the form

$$X(\tau) = e^{\tau \widehat{A}} X(0) \tag{7.3}$$

and, more generally,

$$X(n\tau) = e^{n\tau \widehat{A}} X(0) \tag{7.4}$$

for any positive integer n. However, it is not clear how to compute analytically the elements of the matrix  $\widehat{A}$  or, more importantly, the eigenvalues  $\lambda_1$ ,  $\lambda_2$  of  $\widehat{A}$ . For this reason, we are unable to define a proliferation index,  $\mathcal{P}$ , directly in terms of the model's coefficients. If we denote by  $\lambda_1$ ,  $\lambda_2$  the eigenvalues of  $\widehat{A}$ , with  $\operatorname{Re} \lambda_1 \leq \operatorname{Re} \lambda_2$ , then we expect that the DFE will be stable if  $\operatorname{Re} \lambda_2 < 0$ and unstable if  $\operatorname{Re} \lambda_2 > 0$ .

## **Periodic Case: Estimating** *P* or *R*<sub>0</sub>

If

$$A_1 \le A(t) \le A_2$$

and

$$\frac{dX_i}{dt} = A_i X_i, \qquad X_i(0) = X(0)$$

then

$$X_1(t) \le X(t) \le X_2(t)$$

so that

$$e^{n\tau A_1}X(0) \le e^{n\tau \widehat{A}}X(0) \le e^{n\tau A_2}X(0).$$

#### **Simulations: BB In Colombia**

Parameter	Values per day
$\alpha_B$	0.001, [3]
$\beta_B$	0.00061, [3]
$\beta_T$	0.00048, [3]
$\lambda_B$	0.000265 [3, 25]
$\gamma_B$	0.0002999 [3]
$\gamma_T$	0.001609 [3]
p	0.1 [3]
$\mu_B$	0.0002999 [3]
$\mu_T$	0.001609 [3]

As an example, we shall compare the progress of the disease in the case the initial infection begins near the boundary of  $\Omega$  versus the case when the disease starts in the middle of  $\Omega$ . For simplicity, we take  $\Omega$  to be one-dimensional, namely,

$$\Omega = \left\{ 0 \le x \le 1 \right\}.$$

#### **Initial Conditions: BB In Colombia**



FIGURE 1: Initial conditions when initial infection is at x = 0 and x = 0.5.

Indeed, when the infection starts at x = 0,

$$I_B(0) = \int_0^1 i_B(x,0) dx = \int_0^1 0.5184 * e^{-\frac{w^2}{0.001}} dx = 0.0145,$$
$$I_T(0) = \int_0^1 i_T(x,0) dx = \int_0^1 0.60 * e^{-\frac{w^2}{0.001}} dx = 0.0168,$$

and when the infection starts at x = 0.5,

$$I_B(0) = \int_0^1 i_B(x,0) dx = \int_0^1 0.2592 * e^{-\frac{(w-0.5)^2}{0.001}} dx = 0.0145,$$
$$I_T(0) = \int_0^1 i_T(x,0) dx = \int_0^1 0.30 * e^{-\frac{(w-0.5)^2}{0.001}} dx = 0.0168.$$

Figure 1 highlights the initial BB infections in cattle and tick populations when the initial infection starts at x = 0 and x = 0.5 of  $\Omega$ .

We also take

$$q = 0, \alpha = 10^{-4}, d_B = 0.1, d_T = 0.01.$$

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#### **Profile Of BB in Cattle**



FIGURE 2: Constant Tick Birth rate: Profiles of  $i_B(x, t_0)$ , the infected northern Colombia region cattle population, where initial infection is near x = 0 and x = 0.5, and where  $t_0 = 5, 10$  and 50 years.

#### **Profile Of BB in Ticks**



FIGURE 3: Constant Tick Birth rate: Profiles of  $i_T(x, t_0)$ , the infected northern Colombia region cattle population, where initial infection is near x = 0 and x = 0.5, and where  $t_0 = 5, 10$  and 50 years.

### **Boundary Vs. Interior Initial Infections**



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#### **Impact Of Increased Recovery Rates**



FIGURE 3: Constant Tick Birth Rate: In the cattle and tick populations of Colombia, increasing the rate of recovered bovines,  $\lambda_B$ , shifts the system from BB persistence when  $\lambda_B = 0.000265$  per day to BB extinction in 5 years when  $\lambda_B = 1$  per day and all the other parameters are as in Figure 2.

## Impact Of Decreasing Vertical Transmission



FIGURE 6: Constant Tick Birth Rate: In the cattle population of Colombia, increasing the rate of non-vertical transmission, q, leads to smaller numbers of the total population of infected cattle, where q = 0, 0.3, 0.6 and all the other parameters are as in Figures 4-5.

## Impact Of Decreasing Cattle Infectivity Rates



FIGURE 7: Constant Tick Birth Rate: In the cattle population of Colombia, reducing the infectivity rate,  $\beta_B$ , shifts the system from increasing total population of infected cattle when  $\beta_B = 0.00061$  to decreasing total population of infected cattle when  $\beta_B = 0.0001$  and all the other parameters are as in Figures 4-5.

#### **Tick Control Programs and BB Infections**



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#### **BB In Seasonal Environments**

$$\mu_T(t) = a + b \sin 2\pi t$$
,  $N_{T0}(t) = K + L \sin 2\pi t$  and  $N_{B0} = 1$ ,

where

$$a = 0.001609, b = 0.00156, K = 5, L = 1,$$

and keep all the other parameter values fixed as in Figure 4. With this choice of parameters,

$$\mu_T(t+1) = \mu_T(t)$$
 and  $\int_0^1 \mu_T(t) dt = 0.001609$  as in [3],

while

$$N_{T0}(t+1) = N_{T0}(t)$$
 and  $\int_0^1 \frac{N_{T0}(t)}{N_{B0}} dt = 5$  as in Figures 2-6.

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#### **Seasonal BB in Tick Population**



Figure 9: Periodic Tick Birth rate: As time increases,  $I_B(t) = \int_0^{t} i_B(x,t) dx$ , the sum of infected Colombia cattle population increases while  $I_T(t) = \int_0^1 i_T(x,t) dx$ , the total population of infected Colombia tick, fluctuates where initial BB infection is near x = 0 (blue) and near x = 0.5 (red).

#### **Periodic Vs Constant BB**



Figure 10: Periodic versus constant Tick Birth rate: As time increases,  $I_B(t) = \int_0^1 i_B(x,t) dx$ , the sum of infected Colombia cattle population in constant environment equals the corresponding number in periodic

environment, where initial BB infection is near x = 0 (blue) and near x = 0.5 (red).

## **BB Control Strategies**

We now use data from Brazil, Colombia and South Africa to study the effect of the following control strategies on BB incidence.

- Inoculation only;
- Dipping only;
- Combination of inoculation and dipping.

## Inoculation

To control BB, most cattle producers practice an integrated approach that includes tick control by chemical means (acaricides); cattle vaccination with attenuated strains of *B. bovis*, *B. bigemina* or *B. divergens*, and treatment of clinically-ill cattle. We illustrate the results of cattle inoculation using data from Smith [21]. Accordingly, we calculate the risk of BB among susceptible cattle from the inoculation rate by the equation

$$Q = 1 - e^{-ht},$$

where Q is the proportion of susceptible cattle likely to contract infection within t days when exposed to inoculation rate h.

#### **Brazil Inoculation Data**

Using data from Campo Grande, Brazil, Smith calculated a range for h, h = 0.0009 - 0.0605 per day with mean  $\overline{h} = 0.0123$ . To adopt control program 1 and use it to study the response of inoculation in Model (3.3)-(3.6) with seasonal fluctuations in the growth rate of ticks,  $\mu_T$ , we replace  $\beta_B$  by

$$\overline{Q}\beta_B = \left(1 - e^{-0.0123t}\right)\beta_B,\tag{10.1}$$

and keep all the other parameter values fixed at their current values in Figure 9. Figure 11 shows that after each year of BB, independently of whether the infection starts at x = 0 or x = 0.5, the proportion of BB infected total cattle population under control program 1 is smaller than the corresponding number of infected cattle under the no control program of Figure 9.

## BB Control Strategies: Inoculation or Dipping?



initial BB infection at x = 0 and x = 0.5, using the data in Figure 9, with control programs 1, 2, 3 and no control. Notice that the vertical scale of the graph on the left (x = 0) is extended to show more fully the distinct results of interventions.

## Conclusion

- Our BB model, based on a system of partial differential equations, includes dispersion of both cattle and ticks.
- We introduced the concept of "proliferation index", P, and proved that if P<1 then the DFE is asymptotically stable, whereas if P>1 then the BB will become endemic. Thus, P plays the same role as the more familiar basic reproduction number, R<sub>0</sub>.
- The explicit formula that we derived for *P* shows that *P* decreases as the dispersion coefficients increase. Thus, enhanced dispersion decreases the probability of BB prevalence arising from initial small infection in healthy cattle.

# **Conclusion (Continued)**

- There are several advantages of the PDE model over the ODE model. One of them is that the PDE model shows that the disease is less likely to become prevalent if the dispersion coefficient is increased.
- Another advantage of the PDE model is that it allows us to study how the location of the initial infection affects the progression of the disease. For example, we have shown that when an initial BB infections start at the boundary region, the total population of cattle infections is larger than the corresponding population when the initial infections start in the interior of the region. This has important implications for effective BB control programs.
- We considered also other factors that may affect the progression of the disease. For example, we showed that seasonal fluctuations do not affect the population of BB infected cattle over the entire season.
- We have also quantified the effect of increased number of ticks on the population of BB infected bovine.

# **Conclusion (Continued)**

- Using the value of h of Smith, but adjusting it to the data of Aranda Lozano from Colombia, we computed the reduction in the growth of BB achieved by inoculation, over a period of 10 years. We found that although this reduction is significant, it falls quite short of eradicating BB.
- We considered a second intervention method: dipping cattle in acaricides. This method of intervention was studied by Rikhotso et al. using data from four communal grazing fields in South Africa to analyze the impact of intensive and strategic dipping systems on BB enzootic stability. We found that our dipping program leads to a smaller number of infected bovine than the inoculation program.
- Our model shows that combined inoculation and dipping stabilizes the disease, and even decreases the number of infected bovines, but does not quite eradicate it. It thus suggests that additional control steps need to be taken, for example, aggressive treatment of sick cattle (to increase recovery rate and reduce BB transmission) or a better age-dependent inoculation program.

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# Thank You...

