Mathematical Epidemiology: Going beyond Statistical Mechanics (apparently…)

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Control of endemic SIR diseases through mandatory vaccination

$$S' = \mu(1-p) - \mu S - \beta SI$$

$$I' = \beta SI - (\mu + \nu)I$$

DISEASE-FREE EQUILIBRIUM

DFE = (1-p, 0)

MAY-ANDERSON THRESHOLD

$$p_c = 1 - BRN^{-1}$$

$$p \geq p_c \Rightarrow DFE \text{ is Globally Attractive} \Rightarrow \text{Disease Eradicated}$$

$$p < p_c \Rightarrow DFE \text{ is UNSTABLE} \Rightarrow \text{Endemic Disease}$$

EE = (1/BRN, Ie(p)) GLOBALLY STABLE

Intrinsic limitations of "Classical" Epidemic Models

- **The SIR model is inspired to chemistry = humans are approximated as molecules in a chemical reaction into a (well-stirred) thank !**

«Statistical physics» of Epidemics

$$S + I \xrightarrow{\beta} I$$

# of infections per time unit depends on # of "collisions" => Modeled by Law of Mass Action

$$\frac{dS}{dt} = -\beta SI + \ldots - \ldots, \frac{dI}{dt} = \beta SI - \ldots$$

SIR model = Particles-based model

Human beings are NOT passive particles.

They are <u>active</u> particles endowed with a "psychology".

Capasso and Serio (1978) first stressed that models must face this challenge

"Particles" involved in epidemic models…

WHAT IF THE VACCINATION IS NO MORE MANDATORY ?

$p = p(t)$

?

A.d'O, Manfredi, Salinelli Theor Pop Biol (2007)
A.d'O, Manfredi, Salinelli Math Med Bio (2009)
A.d'O, Manfredi, Salinelli (2013)

PSEUDO-Rational exemption

- Many families base their decisions to vaccinate on informations and **rumours** on:
  - the epidemics: vaccinating more, when many news on disease and little otherwise.
  - available information/rumours on safety of vaccine

"(PSEUDO) Rational" exemption to vaccination

- Myopically parents compare the **perceived** low risk of getting disease and the (rumors-based..) **perceived**risk of vaccine side effects.
- BUT the disease level is currently low **thanks to** large levels of vaccination !!

Simple Phenomenic Model

$$p(t) = p(M)$$

"NEW ENTRIES"

- **M= Information index (?)**
- **p(M) = (information-dependent) vaccination rate at birth**
- **P'(M)>0**

Two equilibria:

$$DFE = (1 - p(0), 0)$$

+ ENDEMIC «BEHAVIOR DEPENDENT» EQUILIBRIUM

The information index M (1)

The case of "current" information:

$$M(t) = g(S(t), I(t))$$

$$\frac{\partial g}{\partial I} > 0$$

Examples:

$g(S,I) = \alpha\beta SI$ (new cases per time unit)

$g(S,I) = kI$ (current cases)

The information index M (3)

Information dependent on past history through delay Kernel K(t):

$$M = \int_{-\infty}^{t} g(S(\tau), I(\tau)) K(t-\tau) d\tau$$

***K(x)=Dirac(x)* => no delay case.**

K(x)= a exp(-ax)

Main questions

Effects of information-dependent vaccination ??

- **Could information-dependent vaccination allow diseases eradication even if the baseline policy (p_0) would not ?** **ANSWER: NO**
- **Does «memory»/delay affects the system stability ?** **ANSWER: YES**

p = p(t)

WHAT ABOUT THE DYNAMICS OF p(t) ?

Bauch 05. d'Onofrio et al 2011, 2015: evolutionary game approach

Word of Mouth: switch of ideas after exchange of infos

Parents behave as total population

P : pro vaccine; A = against vaccine

$$P' = cPA - dPA$$

$$A' = -cPA + dPA$$

$$A + P = 1 \Rightarrow P' = (c - d)P(1 - P)$$

The rates of change: C and D

$C = C(M)$ with $C'(M) > 0$

D : increasing function of information on vaccine side effects !

$d = d(p)$

D'ONOFRIO ET AL 2011

D(P)=k*P

(k = "imitation speed")

C(M)=k*h(M)

$$p' = k\left(h(M) - p\right)p(1-p)$$

$$\text{NOTE THAT}: k >> 1 \Rightarrow p(t) \approx h(M)$$

Another Choice: Bauch 05

$$C = \theta I$$

$$D : constant$$

ERADICATION: impossible

IMITATION SPEED
K
BIFURCATION
PARAMETER

Endemic Equilibrium
INDEPENDENT OF K !

Role of imitation Speed K

- **BAUCH: oscillations for K over a threshold**
- **D'ON: oscillations for K in an intermediate range + sufficiently steep h(I). If also delayed information: CHAOS**

POSSIBLE ACTION OF NATIONAL HEALTH SYSTEM: INDUCING PEOPLE TO VACCINATE

d'Onofrio et al (2012)

INDUCING PEOPLE TO VACCINATE

$$P' = cPA - dPA + G(t)A$$

$$A' = -cPA + dPA - G(t)A$$

$$A + P = 1$$

$$\Rightarrow P' = (c - d)P(1 - P) + G(t)(1 - P)$$

INCLUDING SPONTANEOUS CHANGES (d'Onofrio and Manfredi, 2015)

$$P' = cPA - dPA + j(M)A - b(P)P$$

$$A' = -cPA + dPA - j(M)A + b(P)P$$

$$A + P = 1$$

$$\Rightarrow P' = (c-d)P(1-P) + j(M)(1-P) - b(P)P$$

Eradication: POSSIBLE !!

$$p'(t) = k\,(h(I) - p(t))\,p(t)(1 - p(t)) + G(t)(1 - p(t))$$

$$G(t) > kp_C^2 \Rightarrow I(t) \longrightarrow 0$$

Buonomo & d'Onofrio & Manfredi: Optimal Control

Aknowledgements

- Many thanks to Mirek, Urszula, Monika and Zuzanna for inviting me !
- To you for the patience !

“That’s all Folks !”

(B. Bunny and co.)